**Course description and syllabus**

## General information

**Instructors:** Prof. Chris Wendl (lectures)
HU Institut für Mathematik (Rudower Chaussee 25), room 1.301
`wendl@math.hu-berlin.de`
Office hour: Tuesdays 14:00–15:00 via Zoom
(see `www.mathematik.hu-berlin.de/~wendl/index.html#Sprechstunde`)

Dr. Shubham Dwivedi (problem sessions)
HU Institut für Mathematik (Rudower Chaussee 25), room 1.310
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**Website**: `www.mathematik.hu-berlin.de/~wendl/Winter2020/FunkAna/`

**Moodle**: `moodle.hu-berlin.de/enrol/index.php?id=99581`

**Lectures**: Tuesdays 15:15–16:45
Thursdays 13:15–14:45

**Problem sessions**: Thursdays 15:15–16:45

*The course will be conducted online via Zoom. Links for the Zoom meetings will be made available on the moodle for this course shortly before the start of the semester.*

**Language**: This course is counted as a *BMS Basic Course*, thus it is offered in English unless everyone would like to hear it in German. This will be decided in the first lecture. If you definitely want to hear the course in English but cannot make it on time to the first lecture, please contact the instructor in advance.

**Prerequisites**: The contents of the HU's courses *Analysis I–III* and *Lineare Algebra und Analytische Geometrie I–II*, as well as the analytical part of the course *Algebra und Funktionentheorie* (i.e. basic complex analysis).

All students should in particular be familiar with the fundamentals of measure theory (including Fubini's theorem and the completeness of the $L^p$ spaces).

## Course description

This is a course on *linear* functional analysis, which can be defined as the study of continuous linear maps between infinite-dimensional topological vector spaces (mainly Banach and Hilbert spaces). The most important examples of such infinite-dimensional vector spaces are function spaces, which often arise in applications e.g. as solution spaces for partial differential equations. The contents of this course should therefore be seen as essential preparation for any course (either in analysis, applied mathematics, differential geometry or mathematical physics) dealing with PDEs.

## Syllabus

The course will be divided into three segments:

I. **Fundamentals** (weeks 1–3)

II. **Real analysis and $L^p$-spaces** (weeks 4–9)

III. **Abstract functional analysis** (weeks 10–15)

The following week-by-week schedule is preliminary and subject to change.

1. Basic notions from point-set topology, topological vector spaces, locally convex vector spaces, Fréchet, Banach and Hilbert spaces, examples
2. Continuous/bounded linear operators between Banach spaces, the operator norm, dual spaces, Zorn's lemma, Hamel bases
3. Basic results on Hilbert spaces: uniform convexity, the Riesz representation theorem, orthonormal bases, orthogonal projections
4. Properties of the $L^p$ spaces on $\mathbb{R}^n$: duality of $L^p$ and $L^q$ for $\frac{1}{p} + \frac{1}{q} = 1$, Separability of $L^p$, weak convergence, the Banach-Alaoglu theorem
5. Convolutions and Young's inequality, approximation by smooth functions, absolute continuity, the Radon-Nikodym theorem, the fundamental theorem of calculus
6. Periodic functions and Fourier series on $L^2(\mathbb{T}^n)$
7. The Fourier transform on Schwartz space and $L^2(\mathbb{R}^n)$
8. The Sobolev spaces $H^k(\mathbb{R}^n)$ and $H^k(\mathbb{T}^n)$
9. Distributions (generalized functions)
10. The Baire category theorem and Hahn-Banach theorem
11. The open mapping theorem, closed subspaces with closed complements
12. Compact operators and Fredholm operators
13. The spectrum of a bounded linear operator on a Hilbert space, polar decomposition
14. Spectral theory for bounded operators
15. Unbounded self-adjoint operators and spectral theory

## Literature

For the second segment of the course (real analysis), there will be typed lecture notes (in English) made available on the course website. Otherwise, the course will not follow any particular book, but the following textbooks are highly recommended, especially the book by Reed and Simon.

- Reed and Simon, *Methods of Modern Mathematical Physics I, Functional Analysis*, revised and enlarged edition, Elsevier 2011
  (online access available via the HU library)
- Bühler and Salamon, *Functional Analysis*, AMS 2018
  (preprint version available freely on Salamon's homepage:
  `https://people.math.ethz.ch/~salamon/PREPRINTS/funcana-ams.pdf`)
- Conway, *A Course in Functional Analysis*, Springer 1985
  (online access available via the HU library)

## Exam and problem sets

Grades in this course will be determined by a 3-hour **written exam** soon after the end of the semester (with a resit option shortly before the beginning of the following semester). This will take the form of an electronic "take-home" exam to be written within a fixed time period, for which you may use arbitrary aids such as books and notes, but may not communicate with other students. The exam problems will be conceived so as to be solvable within 2 hours, i.e. time pressure should not be the decisive factor.

**Problem sets** will be posted on the course website every Thursday, and solutions can be submitted via the moodle any time until the following Thursday (the solutions will be discussed in the problem session).

There will also be a special homework assignment midway through the semester, the so-called **take-home midterm**, which you will have two weeks to work on.

The problem sets and midterm are voluntary, but your scores on these assignments can be used to boost your final exam grade according to the following rule:

- Problem sets $\geq 50\%$ **or** midterm $\geq 75\%$ $\quad\Rightarrow\quad 2,0 \rightsquigarrow 1,7$ or $1,7 \rightsquigarrow 1,3$ etc.
- Problem sets $\geq 50\%$ **and** midterm $\geq 75\%$ $\quad\Rightarrow\quad 2,0 \rightsquigarrow 1,3$ or $1,7 \rightsquigarrow 1,0$ etc.