Berlin Trails to Max-Planck-Institut and to the Fields Medal 2018

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Abstract

The co-director P.Scholze of the MPI (Max-Planck-Institute) for mathematics (Bonn) received the Fields Medal on the ICM (International Congress of Mathematics) 2018 in Rio de Janeiro. It's the highest international prize for mathematicians, comparable with the Nobel prize in other fields. It was the second time that the honour has been dedicated to a German mathematician. The first prize-winner in Germany was Gerd Faltings (in 1986). Ten years later he holded the position of a director of the MPI. Personally I met him at some conferences around 1983 and also later. In the 90-th changed the Russian mathematicion Juri Manin took a co- director position in Bonns MPI. He gave me already in GDR-times in Berlin and Moscow most valuable hints for my own mathematical work leading e.g. to the first Euler Lecture (1982). It was organized by the Karl-Weierstrass-Institute of the Academy of Science of GDR and took place in the Academy-Building near the Gendarmenmarkt ("Place of Academy" at this time) in Berlin-Mitte.

An important role played also the Max-Planck-Gymnasium in East-Berlin as well for Peter Scholze as (much earlier) also for his supervisor Michael Rapoport. This gymnasium became a direction to special mathematical teaching and learning in the 1960-s. The main initiators of this special education were my mathematical teachers

Figure 1: Felix Mendelssohn-Bartholdy (1809-1847)

at the Humboldt-University Prof.Dr. Hans Reichardt and Prof.Dr. Heinrich Grell. They selves learned Mathematics during the 1920/30-s from great mathemations of the last century. Here is a list of names:
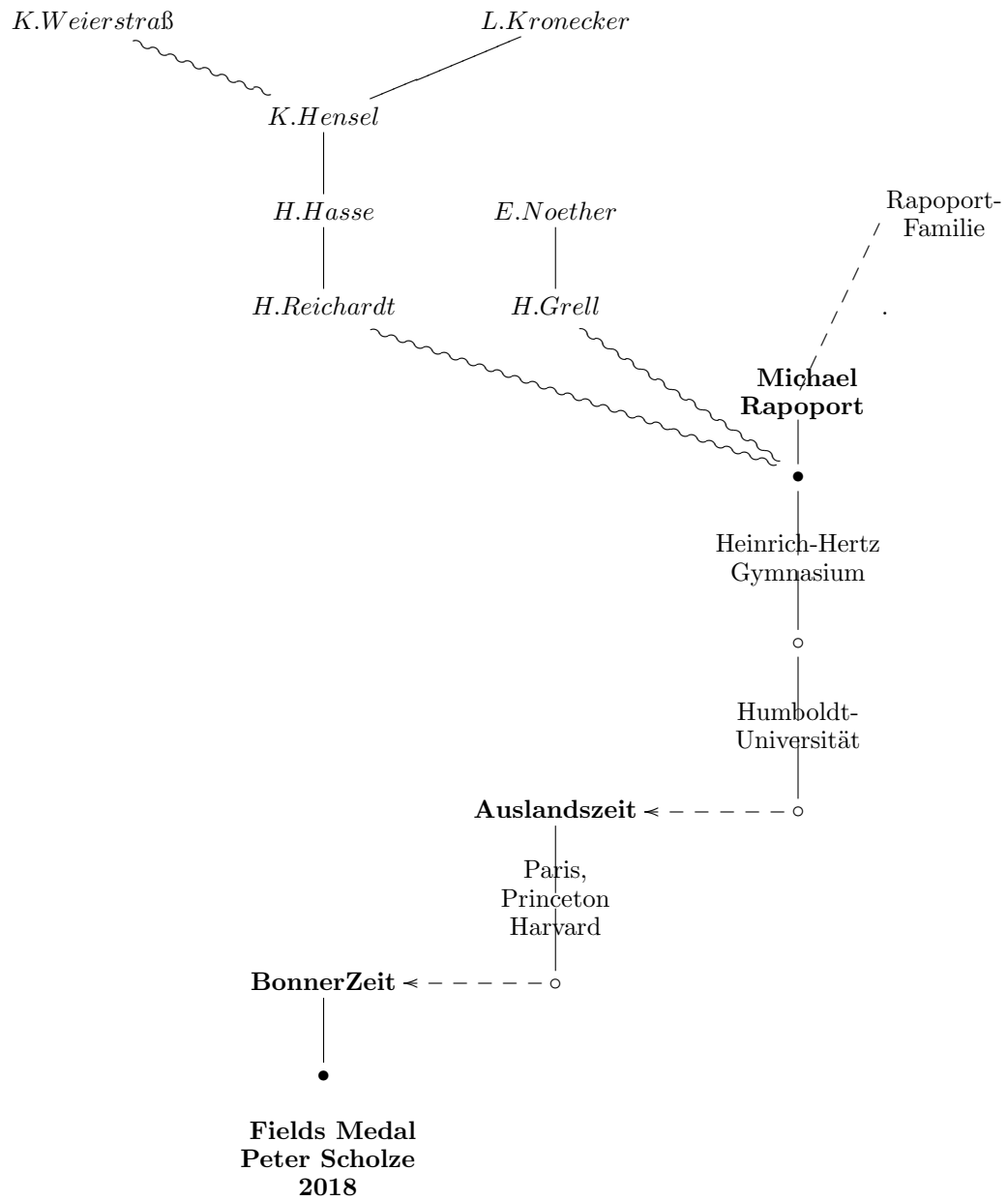E. Artin, E. Hecke, F. Klein, Hilbert, P.Gordan, I. Schur, E. Noether, H. Hasse, M. Deuring, C.L. Siegel, F.K. Schmidt, B.L. van der Warden.
It is basical for the great energy and power of my teachers for the development of our special mathematical school preparing pupils to world-wide successful mathematicians.

Contents

1 Intro (Historirical Developments)

K.Weierstraß   *L.Kronecker*

K.Hensel

H.Hasse   *E.Noether*   Rapoport-
Familie

H.Reichardt   *H.Grell*   .

**Michael
Rapoport**

Heinrich-Hertz
Gymnasium

Humboldt-
Universität

Auslandszeit ⊰ − − − − − ∘

Paris,
Princeton
Harvard

BonnerZeit ⊰ − − − − − ∘
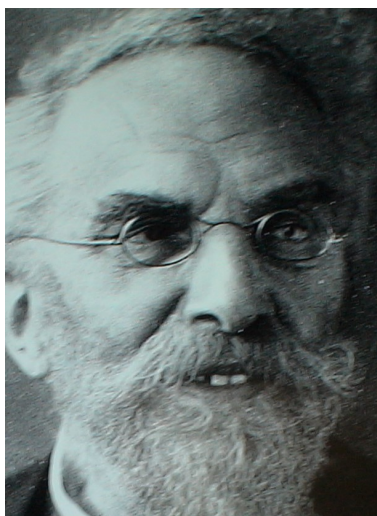
Fields Medal
Peter Scholze
2018

Figure 2: Kurt Hensel (1861-1941)

He stems from a famous family dynasty: Fanny Mendelssohn-Bartholdy (1805-1847) was his grandmother of his father's side. To K.Hensel's direct forefathers belongs Moses Mendelssohn (1729 - 1786). The well-known music composer Felix Mendelssohn-Bartholdy was the grand-uncle (sister of Fanny) of K.Hensel.

2   Kurt Hensel

Short Biography

K.Hensel was born in Königsberg. Until his nin-th year he was educated by his parents at home. Then the family moved to Berlin, because Kurt's father took there the position as director of the German Construction Company. K.Hensel received now an excellent education in mathematics and physsics at Dbberlin's Private School, called Friedrich-Wilhelms-Gymnasium at later times. Hensel began to study in Bonn, mainly attending lectures of Lipschitz. After two years he changed to Berlin, where he heard lectures of Weiersrass and Kronecker. His main interest turned to number theory. Consequently, L.Kronecker was the advisor of his dissertation entitled "Arithmetical studies on discriminants and their extraordinary divisors". He began in 1886 to work on his postdoctoral thesis (ha-
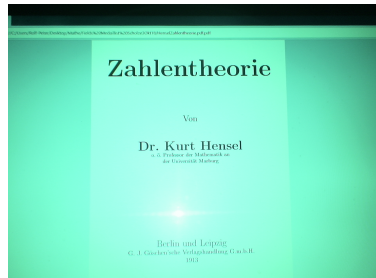
Figure 3: Zahlentheorie Lehrbuch, Hensel 1913

bilitation), again working on Kronecker's side. Inbetween Hensel volonteered one year ago at the army (in Freiburg). He began to work Berlins University as lecturer (Privatdozent), and from 1891 he has been promoted to extraordinary professor. Ca. ten years later he followed the call of the Marburg University to an ordinariat. There Hensel worked from 1902 till retirement (1930) and lived there till the end of his life. In the year of 1917 he was chairman of the DFG (German Mathematics Society).

Personal Relations:

Already after his promotion Hensel got engaged secretly to Gertrude Hahn. After his habilitation this event was publicy announced. A half year later they got married. Their marriage lasted till the end of life. Four daughters and a son issued from this union. All grandchilds of Moses Mendelssohn had been converted to Protestantism. But from the Nazis classifyed as Jews Hensel's family were more and more repressed since 1933. The "Law of restoration of civil service" could not directly applied to Hensel because of his emeritus status. Nevertheless Hensel's last years were shaped by some of related sorrows. He was cancelled from the list of members of the world-wide respected Leopoldina Academy. It's not clear whether he was informed and/or aware about this procedure. K.Hensel died 1941 after a heart attack.

Scientific Outputs

Hensel's most influencing contribution to mathematics was his creation of p-adic numbers. They growthed to a basic tool in number theory. It was essentially used by our Fields Medalist P.Scholze in

his work. Hensel joined ideas of Kronecker and Weierstrass. He tried to apply results of function theory to the theory of numbers. In order to solve open divisibility problems Hensel consider Laurent series in p-powers

$$a = \sum_{-\infty \ll i < \infty} a_i p^i, \quad a_i \in \mathfrak{R}_p \tag{1}$$

of rational numbers a, p a fixed natural prime number[1]. Thereby $\mathfrak{R}_p \subset \mathbb{Z}$ is a fixed complete system of residues modulo p. Usually one takes the smallest natural one $\{0, 1, 2, ...p-1\}$. Shortly, any series in (1) is called a *p-adic* one. The set of all of them is denoted by $\mathbb{Q}_p$. It is a field, called *field of p-adic numbers*. The subring of all Taylor series in (1) is called *ring of integral p-adic numbers*. It is denoted by $\mathbb{Z}_p$. Then p is also a prime element in $\mathbb{Z}_p$.

In analogy to the construction of real numbers with help the of Cauchy sequences of rational numbers, can $\mathbb{Q}_p$ be considered as completion of $\mathbb{Q}$ with respect to the *p-adic metric* $|.|_p$ [2]. In function theory the Laurent series one knows Laurent series

$$f = \sum_{-\infty \ll i < \infty} \alpha_i (z - z_0)^i, \quad \alpha_i \in \mathbb{C} \tag{2}$$

for complex functions $f(z)$ with (at most) a pole in z_0 and holomorphic in a neibourhood. Here $z - z_0$ plays the role of prime element in the ring of (convergent) Taylor series in (2).

Remark 1 *The ideas to analogies between function and number fields go back to Richard Dedekind (1831-1916)und Rudolf Heinrich Weber (1874-1920). A common language has been first found by the German algebraist Wolfgang Krull (1899-1971). In the 50/60-s things came completely together by the scheme language in algebraic geometry introduced and investigated by Alexander Grothendieck (1928-2014).*

The first progress of p-adic studies was the following result of Hensel himself. Consider the reduction maps ($g \mapsto g \mod p$)

$$red_p : \ \mathbb{Z} \to \mathbb{F}_p \ := \mathbb{Z}/p\mathbb{Z} \text{ and } \mathbb{Z}_p \to \mathbb{F}_p = \mathbb{Z}_p/p\mathbb{Z}_p$$

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[1] $\ll$ bedeutet, dass i höchstens endlich viele negative Werte in der Summe annimmt.
[2] $\mathbb{Q} \ni q = \frac{a}{b} p^k \mapsto p^{-k} =: |q|_p$, where $k \in \mathbb{Z}$, $p \nmid a, b \in \mathbb{Z}$

and their extensions to the polynomial rings (sending X to itself)

$$\mathbb{Z}[X] \to \mathbb{F}_p[X] \text{ and } \mathbb{Z}_p[X] \to \mathbb{F}_p[X]$$

also denoted by red_p.

Lemma of Hensel. Assume that the p-reduction $f \in \mathbb{F}_p[X]$ of a normed polynomial $F(X) \in \mathbb{Z}_p[X]$ is product of two non-constant coprime normed polynomials g and h. Then also F is product of two normed polynomials $G, H \in \mathbb{Z}_p[X]$ with reductions g, h, respectively.

This lemma is usually teached in first courses of number theory at universities. it has a simple surprising application: It reduces the problem of the existence of a $\mathbb{Z}_p$-zero of a polynomial $F \in \mathbb{Z}_p[X]$, as in Hensel's lemma, to a finite problem looking at the reduction polynomial f. One has only to find a linear factor of f in $\mathbb{F}_p[X]$, same to say, an $\mathbb{F}_p$-zero of f. One has to check only the finite elements of $\mathbb{F}_p$ for get a zero or none.

More acceptance in mathematics got the p-adic numbers but after Hensel's doctorand Helmut Hasse pushed forward from p-adic numbers to diophantine equations over $\mathbb{Q}$. Namely, in his doctoral thesis he gave an idea of the *local-global principle*. As guideline he formulated and proved for quadratic forms over $\mathbb{Q}$ the p-adic criterion for the existence of a non-trivial rational zero, see below refP1.

My touch points

These are only of second hand kind. My teacher of number theory at the Humboldt- University Prof.Dr. Hans Reichardt told us from the lectures he heard in Marburg by Hensel. They were accompanied by anecdotes about other persons. Once another student asked him whether the stories happened really in that manner as he had told them. Hensel answered: "I tell the things anytimes in such a way as they should be happened, but not how they occured by chance".

Reichardt told us also, that Hensel has been considered as rather quirky with his series with ascending p-powers. For myself, I was encouraged to read Hensel's book "Number Theory" of 1913. This was done during my summer vacations after my 6-th semester on the beaches of Dahme and Spree in Berlin. With pleasure I followed the clear constructions till his explanations of p-adic rules.
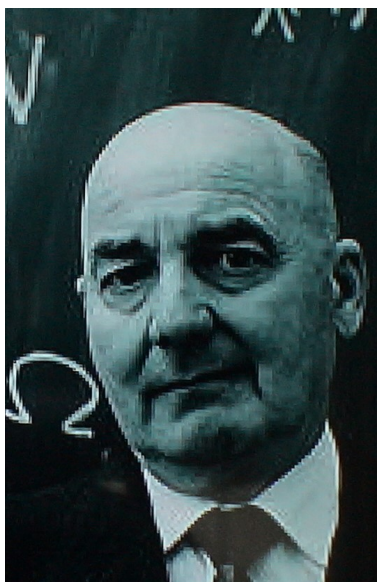
Figure 4: Helmut Hasse, 1898-1979

3 Helmut Hasse

Short Biography

H. Hasse was born in Kassel. He finished the school in the year of 1915 with an auxiliary abitur. In 1918 he began to study at the University of Göttingen. He heard lectures of Edmund Landau, David Hilbert, Emmy Noether and Erich Hecke.

He completed his study from 1920 on in Marburg under emphasized influence of Hensel. Hasse familiarized hisself with number theory, especially with p-adic numbers. His thesis "On the Representation of Numbers by Quadratic Forms in the Field of the Rationals" extended successfully the applications of p-adic numbers. Further extensions were written in his habilitation article "On the Equivalence of Quadratic Forms in the Field of Rational Numbers".

In the winter semester of 1922/23 he worked as lecturer at the University of Kiel, not far from Hamburg. On this line he found fruitful contacts with Artin, Ostrowski, Petersson and Schreier. In the year 1925 Hasse followed the call of a professorship in Halle. After Hensel reached the emeritus status in 1930, Hasse took up the place of his earlier main teacher in Marburg.

Four years later he received the vocation to the director of the Mathematical Institute of Göttingen. Much scientific positions were free in Germany by the aggressive laws of Nazis against Jews. During the years of war Hasse leaded a Research Institute for ballistic problems.

Shortly after the war he could not teach again at a university because of his (rather inactive) membership in the Nazi party NS-DAP and a decision of british occupying forces. So, in the year of 1946 Hasse changed to the Soviet Sector of Berlin. There he extended his number theoretic research work at the new-founded German Academy of Sciences in Berlin-Adlershof. Only in 1949 he took up again his teaching work, this time on the Humboldt University of (East-)Berlin. Then (1950) he followed the call to Hamburg, where he remained till the end of his life. But he maintained further friendly contacts with East-Berlins Academy of Sciences.

After emeritation (1966) he gave several lectures foreign contries, e.g. at the Hawaii-University. Helmut Hasse died 1979 in Hamburg.

Scientific Outputs

The following result is known as

Theorem of Hasse.
Let
$$f(x_1, x_2, ..., x_n) = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} x_i x_j$$

be a quadratic form with coefficients a_{ij} in a number field K. Then from the existence of non-trivial zeros in $\mathbb{R}$ and in all p-adic completions of K follows the existence of a non-trivial K-solution.

Remark. First this theorem has been proved for the field $K = \mathbb{Q}$ of rational numbers. This starting case is known as **Satz von Hasse-Minkowski**.

For his habilitation Hasse proved that two quadratic forms over $\mathbb{Q}$ are equivalent (over $\mathbb{Q}$) if and only if they are equivalent for all p-adic fields (over $\mathbb{Q}_p$) and also over the reals.

The conclusion from local fields (p-adic and real ones) to the global field (K in the theorem or $\mathbb{Q}$ in the remark) is called *Local-Global-Principle*. If the conclusion in a local-global comparasion is true, then one says: The *Local-Global-Principle* is valid.
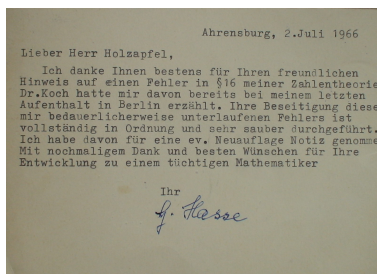
Figure 5: Words of thanks, 1966

Animated by David Hilbert, Hasse wrote his celebrated "Class Field Report" ("Klassenkörperbericht"), compiling earlier results of Kronecker, Weber, Hilbert and adding several valuable new ones.

Let E be an elliptic curve over $\mathbb{Q}$ with conductor[3] N and $\zeta(s)$ the Riemann Zeta-function. The *Hasse-Weil Zeta-Function* of E is explained as

$$Z_{E,\mathbb{Q}}(s) = \frac{\zeta(s)\zeta(s-1)}{L(s,E)}$$

Thereby is $L(s,E) = \prod_p L_p(s,E)^{-1}$ the *L*-Funktion of $E/\mathbb{Q}$, with local factors

$$L_p(s) = \begin{cases} 1, & if\ p^2|N \\ (1-a_p p^{-s}) & if\ p|N,\ p^2 \nmid N \\ (1-a_p p^{-s}+p^{1-2s}) & if\ p \nmid N \end{cases}$$

with $a_p = p+1$ if E has good reduction at p. Otherwise $a_p = \pm 1$, the sign $+$ must be chosen iff E has splitting multiplicative reduction[4].

The *Hasse-Weil Conjecture* states that the above Zeta-Function of any elliptic curve E can be extended to a meromorphic function on the whole s-plane with help of a functional equation. For semistable elliptic curves the conjecture has been proved by Adré Weil in connection with the proof of Fermat's Last Theorem . The general case was proved only in the year 2001 by C.Breuil, B.Conrad, F.Diamond and R.Taylor as consequence of the celebrated *Moduli Theorem*. The Moduli Theorem (conjectured by Shimura-Taniyama in the 50-s) is a (proved) special case and important starting point

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[3]the conductor will not be explained in this paper
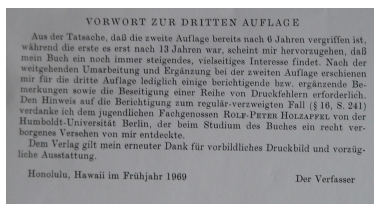[4]https://en.wikipedia.org/wiki/Hasse%E2%80%93Weil_zeta_function

Figure 6: Vorwort zur 3.Auflage der Zahlentheorie von H.Hasse

for the extensive *Langlands-Program*[5].

My Touch Points

In the framework of the Number Theory Seminar 1964/65 the members were invited by the organizing head Prof. H.Reichardt to the Institute of Mathematics and Mechanics of Academy in Berlin-Adlershof to an interesting meeting with H.Hasse. We just studied in our Seminar at the Humboldt University Hasse's famous textbook "Zahlentheorie". A littlebit later I discovered in his book an inaccuracy: a non-valid formula. Reichard encouraged me to communicate this fact to Hasse in Hamburg. I wrote it together with the correcture to H.Hasse. The correct formula was also my first mathematical publication: "A Remark to Hasse's Number Theory". Surprisingly, I received a nice reply card from Ahrensburg (near Hamburg), see Figure 5. Also H.Reichardt was a littlebit proud about his school from where the correcture came. In the 3-rd edition of the Zahlentheorie-Book Hasse mentioned my name in the new preface, see Figure 6.

4   Hans Reichardt

Kurze Biography

Hans Reichardt was born in Altenburg (Thüringen) as son of a doctor med. He visited the Humanistic Gymnasium in Altenburg and later, from 1926 on, the universities of Königsberg, Berlin, Hamburg and Marburg. In Berlin (1928) he heard the lecture "Algebraic Number Theory" by Issai Schur. In Hamburg he visited teaching sessions of Erich Hecke and Emil Artin. In Marburg Reichardt was

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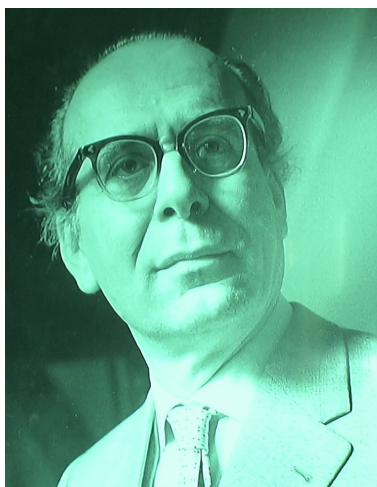[5]https://en.wikipedia.org/wiki/Modularity_theorem

Figure 7: Hans Reichardt, 1908-1991

besides of mathematics also interested on physiks and philosophy. He obtained a doctorate under H.Hasse with the work "Arithmetic Theory of Cubic Fields as Radical Fields".

In the year 1934 he took the teaching degree state examination, changed then as assistent to the University of Frankfurt on the side of Carl Ludwig Siegel, then (1935) to F.K.Schmidt in Leipzig and finally to B.L.van der Waerden at the University of Leipzig. There he habilitated 1939 with the theme

$$\text{On the diophantine equation } ax + bx^2y^2 + cy = ez^2$$

He took in 1940 there a lectureship. At war time Reichardt, from 1943 till the end, worked in the Telefunken corporation in Berlin.

After the war Reichardt had been posted (1946) to the Soviet Union. There he worked on Rocket Technology on the island Gorodomlja (today: Settlement Solnetschny) in the Seliger Sea (Wolga headwaters). In the year 1952 he returned to Berlin and received a chair at the Humboldt-University. In 1955 he took possession of director of the of the I. Mathematical Institute. From 1959 on Reichardt advanced to the director of the Institute of Pure Mathematics of the German Academy of Sciences in Berlin-Adlershof, where he also managed the research group "Number Theory". He reached the emeritus stage in 1973.

Scientific and Organisatoric Efficiencies

He showed in 1942 that the Hasse-Principle is not valid generally for elliptic curves over $\mathbb{Q}$. He presented an example without any $\mathbb{Q}$-rational point, but whose $\mathbb{Q}$-equation has p-adic non-trivial solutions for each prime number p.

In the 1930-s Reichardt attended to the Inverse Problem of Galois Theory (which looks for a Galois-field realization of a given group). He solved the problem for p-groups in positive sense (if p odd). The work had been intensively extended by his Russian collegues Dmitri K. Faddejew and Igor R. Shafarevich in the 1950-s, furthermore in Germany by Jürgen Neukirch in the 1970-s.

After the second war Reichardt turned to differential geometry. He built a powerful research group at Humboldt-University in this direction, where the mathematicians Rolf Sulanke und Thomas Friedrich came from. On the other hand Helmut Koch took over successfully the group of Number Theory in the Academy in Berlin-Adlershof. During the second university reform, dividing the instituts into sections, the number theory changed at this time from university to academy. Reichardt and Koch succeeded to join the work closely with Moscow's Shafarevich-Group (Algebraic Geometry and Number Theory) at the Steklov-Institute for Mathematics of the Academy of Sciences of USSR. The members of the Reichard/Koch groups made large profit of this connection. For example, I was invited three times by the world-wide celebrated Shafarevich to give a lecture at the Steklov-Institute. It should be mentioned on this place, that (for political reasons) I.R.Shafarevich was banned from entering the Lomonossow-University.

From the 50-s on Reichardt devoted himself to history, especially to the work of C.F.Gauß. In the year 1974 he wrote a Gauß-article in the famous Encyclopedia Britannica. Also unforgettable is his edition of the commemorative book on the occasion of 100-th anniversary of Gauß' death in 1955 (published by Teubner 1957).

Reichardt was also the main initiator of Mathematical Olympiads in GDR-schools which start in 1960. I remember to my first Reichardt-lecture in autumn 1961, when he presented the first winner as his "Mathe-Grandchild". He took also care especially to us students. So, he took always part on our practice hours. Often he felt tired, closed the eyes, but when sombody on the blackord produced an error, then he woke up suddenly, in order to correct him.

My Touch Points

In my first three student years (1961 - 1964) H.Reichardt was my central mathematics teacher at the Humboldt-University. I visited his lectures in Linear Algebra, Algebra and Number Theory together with accompanying practices and seminars. At the beginning of the third year three of my seminar group were introduced to Reichard as the best ones. He was asked for a special advanced algebra program for us. But he cosidered such program to be premature for us. We should better better learn for broader basis knowledges. This was a good advice. Later we could make a large profit of this decision.

In the third and fourth year I heard the lectures of Number Theory of Reichard. Together with Dr.H.Koch from the Academy he guided the seminar. As I already mentioned at the end of Section 4 I discovered a wrong formula in Hasse's Zahlentheorie. In my seminar talk I had a little struggle with H.Koch. He could not imagine that Hasse made a mistake in his book. But Reichard could believe it. What followed, I mentioned in Section 4.

Reichard invited me to write my diplom thesis extending his own earlier doctoral work in number theory. But at this time I took part on a new seminar in Algebraic Geometry. The organizers, Prof.H.Grell and the algebraic shooting star Dr.L.Budach, came also from the academy. So I asked there for a diplom theme, but took also further part on number theoretic educations beside of algebraic geometric ones. For success, see e.g. Grell's evaluation in Figure 11 below, Section ref6.

Later, Reichard and me came together again, after my assistent time (1969 - 1979) at Section Mathematics of the Humboldt-University and after changing from university to the Mathematical Institute of the Academy of Sciences of GDR (1979).

In the mean time I was delegated to a supplementary study year (1973/74) to Leningrad (today St.Petersburg). Together with two Russian students and collegue from Dresden we lived in one room. Kitchen, bathroom and WC we had to divide with the whole floor. At the university we were evaluated for distributions to advanced scientists by an expert of the universioty. My evaluator was Z.I.Borevich from Number Theory. His name was known to me from the celebrated Russian textbook "Number Theory", which he wrote together with I.R. Shafarevich. After speaking about mathemat-

ics, my interests in algebraic geometry and corresponding knowledges, he told me that no of professors of the university could help me here to further progress. But wanting to help me, he said: I can arrange for you a contact to the Leningrad departement of the Steklov-Institute for Mathematic belonging to the Academy of Sciences of USSR. There works a mathematician of world level: Boris Borissovich Venkov, with broad knowledges in algebraic geometry and touching branches. Aha, this is the man who translated skillfully Hirzebruch's celebrated book "Topological Methods in Algebrai Geometry". Asking for a connection with current Russian algebraic geometrical research, he gave me a fresh dissertation from Moscows Vinberg-School. He was one of the reviewers. Suddenly I was fascinated from the modern interaction of number theory, algebraic geometry, differential geometry and analysis. For an understanding I had to fill a lot of gaps in my knowledges. So I started a race to catch up those through old and new mathematical achievements till I understood the work completely. After some times I was able to develop the ideas further.

After a talk in the Banach Center in Warschau was announced to the GDR-Academy by a member: "This is the best of mathematics done after the war in Berlin". I got from Prof. H.Koch the offer to change from university to the Mathematical Institute of the GDR-Academy. I accepted, and at the same time I submitted my scientific work for the "Doctor of Sciences" (equivalent for habilitation in GDR) at the Humboldt-University as leaving present. Necessary were three reviewer of different branches of mathematics. I asked Prof. H.Koch (Number Theory), Prof. H.Kurke (Algebraic Geometry) and Prof. R.Sulanke (Differential Geometry). The chairman for the defend my work was my old teacher Prof. H.Reichardt.

During a later visit of the MPI-director F.Hirzebruch H.Reichart encouraged me keep close to the former, what I did. After his retirement Reichardt gave me as present a thousend pages thick book: The complete edition of the work of Erich Hecke. At this time I didn't know that I will really need it. But I did, until now, and he knew it. I am very grateful.

Figure 8: Emmy Noether, 1882-1935

5 Emmy Noether

Short Biography

Emmy Noether is without any doubt the most famous woman in mathematics of all times (until now). But she belonged to Göttingen, not to Berlin. Only his doctorand Heinrich Grell came after the war to Germany's capital. That's the reason why we remember E.Noether on this place as a precursor of the Berlin-trail of the last century.

E.Noether was born in Erlangen. She was descended from a well-off Jewish family. Today a plaque in the Erlanger Hauptstraße reminds of her birth house. Her father Max Noether had a chair of mathematics at the university of Erlangen.

In the year of 1903 she brought in Nürnberg the external high-school exam after, precisely on the Royal High School (today: Willstätter Gymnasium). In 1903 Bavarian universities allowed first time women to study there. She was registered at the university of Erlangen in the same year. She received her doctorate from P. Gordan in 1907. In the year 1909 she was called from Felix Klein and David Hilbert to the Georg-August-University in Göttingen. In

this time she was already a well-known mathematician in the research field of differential invariants. Göttingen at that time was considered the leading mathematical center in the world. Encouraged by Klein and Hilbert, Noether applied on 20-th July 1915 for a habilitation in Göttingen. The application was followed by intensive controversial discussions in the faculty. Most of the members in principle spoke out against the habilitation of women. But in the end Hilbert and Klein were able to assert themselves. Renowned in this connection has been the comment of Hilbert: "A faculty is not a bathing establishment". Since the habilitation of women at prußian universities was prohibited by a decree of 29-th May 1908, the mathematical-scientific department of the the Faculty of Philosophy the University at Göttingen on November 26, 1915, made an official request to the Prußian minister. A final negative answer came back in 1917. Emmy Noether was left with nothing but announce her lectures under the name of Hilbert, as whose assistant she acted. After the World War, Emmy Noether became (in 1919) the first woman in Germany habilitating in mathematics. She was also the first woman in Germany to receive a (not civil servant) professorship. It wasn't until 1922 that she was awarded an extraordinary professorship and received her first paid teaching position in 1923.

In the academic year 1928/29 she accepted a visiting professorship in Moscow and 1930 in Frankfurt (Main). In 1933, Emmy Noether was prosecuted by the so-called law of the Nazi regime revokes their teaching permits.

At the end of 1933 E.Noether received a visiting professorship at the Womens College Bryn Mawr in Pennsylvania (USA). From 1934 she also read lectures at the renowned Institute for Advanced Study. There she was strongly influenced by Oscar Zariski. Emmy Noether died on 14-th April 1935 due to the complications of an abdominal surgery that had become necessary due to a tumor.

6   Heinrich Grell

Kurze Biografie

H.Grell was born in Lüdenscheid (NRW) as son of a master butcher. He graduated from high school in his hometown in 1922.

Figure 9: Heinrich Grell, 1898-1974

Afterwards he studied mathematics, physics and astronomy at the University of Göttingen. In 1926 Grell received his doctorate from Emmy Noether with the dissertation "Relationships between the ideals of different rings". With the help of a scholarship of the "Notgemeinschaft Deutsche Wissenschaft" he worked 1927/28 as an auxiliary assistant in G"ottingen. He then worked as an assistant at the University of Jena until 1930. There he wrote his postdoctoral thesis "Ramification Theory in Arbitrary Oders of Algebraic Number- and Function Fields". Afterwards he became a lecturer in Jena for a short time. He then had to reorient himself mathematically, habilitated newly in Halle (1934) and has now become a lecturer for analysis and analytic geometry.

In 1935 Grell he had been arrested for violation of 175 of German's criminal code (homosexual acts) and lost his teaching license. For my opinion this was absolutely incorrect, because I know him as a serious husband. He was unemployed from 1935 to 1939 and kept his head above water with odd jobs.

The war brought him to Messerschmitt Aircraft Works. There he was group leader in the development department until 1944. In the last year of the war he was appointed to the Reich's Research Council, based in Erlangen. After the end of the war he resumed

his work as an assistant, successively at the universities of Erlangen and Bamberg. In 1948 he accepted an appointment at the Humboldt-Universitt of Berlin, to which he belonged until his retirement (1968). In between he moved from West Berlin to the eastern part of the city after 17 June 1953, because at the border crossing on his way from his residence (West-Berlin) to the HU, there were obstructions. He told me later about this.

From 1959 till 1962 H.Grell was the Executive Director of the Institute for Pure Mathematics of of the former German Academy of Sciences in Berlin-Adlershof. In 1964 he was promoted to Deputy General Secretary of the entire Academy. He remained in this position for eight years.

Heinrich Grell died in 1974 in Berlin.

My touching points

From the fourth year of my studies I took part in the well attended seminar "Algebraic Geometry", which was led by H. Grell and the young talented academy researcher Dr. Lothar Budach. At the same time I also attended the Grell lecture "Theorem of Riemann-Roch" (with algebraic reasoning). Impressive for me were the basics of Richard Dedekind and Emmy Noether, that were used, advancing then to modern representations. This was the last lecture of Grell. I also made a little film about a day of hiking in the seminar group to the Stienitz Sea (near to Wandlitz), which I have digitized in the meantime. It contains valuable recordings with Grell, Budach, collegues and students.

At the end of the academic year I asked him to give me a theme for the diploma thesis. He proposed "Ideal Theory in μ-noetherian rings". In addition, I asked for an appointment at the Academy, in order to be employed algebraic-geometrically as an assistant beyond the diploma degree course. Even the heavy entrance door to the room marked "Deputy Secretary General" very much impressed me. It was located in the academy building opposite to the "Gendarmenmarkt" (at this time: "Platz der Akademie"). During the interview, Grell drew attention to a broad basic knowledge is necessary. Especially, I should attain a good degree in "Theoretical Physics" as a minor subject. In addition, he also suggested me to work through the two volumes of "Commutative Algebra" of Zariski-Samuel. I had already bought the two volumes in Russian before.
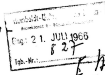
Figure 10: Grell's evaluation for aspirancy (page 1)

The first volume I had already devoured earlier. You could buy them cheaply in the "House of the Socialist Book" near the Neptune Fountain. With vigour I learned now. One year later I was holding the diploma certificate with the overall grade "excellent" in my hands.

Punctually in September 1966 I started then my aspirancy at HU Berlin, Section Mathematicsm according to the plan. "Ideal Theory in Abelian Categories" was the topic of my dissertation, to be written over a period of three years. The Grell-Budach-Seminar continued, where the level was raised significantly with orientation towards the joint study of top international projects in the field of Algebraic Geometry (Grothendieck, Deligne, Shafarevich). I was Grell's last doctorand. The main results of my dissertation flowed into the monograph "Localisations and Grothendieck Categories", written together with L.Budach.

Scientific-Organisational Services

I don't have any scientific (basically new) knowledge of Grell's work in the period after World War II. He was mainly engaged in teaching and organising the reconstruction of a strong mathematics

Figure 11: Lothar Budach, 1935-2007

in Germany. Especially he succeeded in building up a strong school of algebraic geometry in the GDR. Together with the HU-Professor H. Reichardt he initiated in the 1960-s the establishment of a special school with a mathematical-scientific orientation, the first one in Germany. The Heinrich-Hertz-Gymnasium in Berlin-Adlershof was chosen for this purpose. It is still active in this direction until today, after having moved twice within Berlin. Meanwhile, we have a field medal winner who went through this school: Peter Scholze (2018).

We also note that the doctoral mother E. Noether (of Grell) and the doctoral supervisor H. Hasse (of Reichardt) worked closely together around 1930. A scientific correspondence (1925 - 1935) between the two, about deep algebraic problems, has been preserved and is available on the Internet.

7 Budach's Youth Research Group

During my first year as a doctoral student, the seminar was transformed into a vertical learning and research group. The best stu-

Figure 12: MMM Document, 1968

dents from different six years of study (I was counted to the 6-th year) interested on algebraic geometry have been summarised. I got the job to identify the most capable for research. With them, Marko Roczen and Gerhard Pfister, began a common research work with theme: "Primary Decompositions in Abelian Categories". It brought us to Leipzig's MMM (Messe der Meister von Morgen, english: Fair of the Masters of Tomorrow). I remember the visit of the head of the GDR-government Willy Stoph in our exhibition hall and an evening performance by the renowned actress and Brecht interpreter Gisela May. By the way, G. Pfister was appointed professor in the 70-s at the HU, later, after the fall of the Berlin Wall to Kaiserslautern.

Furthermore, three very talented graduates of the Heinrich-Hertz-Gymnasium joined us in the seminar, among them Michael Rapoport. They already were equipped with the mathematical skills of basic studies. In addition to the mathematically oriented curriculum at the Gymnasium they received they received a special grant from Prof. L. Budach. He immediately included them in our group at the university. At her first appearance I was just lecturing about "Étal Topology". It must have been a shock to them. But they stayed tuned, passing us one by one and two of them later became

professors in the old Federal States of Germany. They have been one of the world's best since that time: Michael Rapoport and Thomas Zink.

To get funds for autumn schools and participation in conferences, the Grell-Budach research team has been declared as FDJ-group (FDJ: Freie Deutsche Jugend, english: Free German Youth; mass organization of GDR). As political theme we dealt with "Marxist Penetration of Mathematics". Mainly we discussed thereby philosophical questions of mathematics, on which our science is not poor in, as one knows.

A group organizer had to be found among us students. Since I was the most advanced of the group, I was suggested for this job. I noticed I'm not a member of the FDJ organization. "It doesn't matter, no one notices" but I've been replied, and whoosh, I was elected. Wisely, I have concealed my previous problems with the FDJ. Namely my parents didn't allow me to take part on Pioneer Organization and FDJ. There existed also such a kind of dictatorship in the GDR occasionally. I had to take a detour to get my high school diploma and university degree. But all right, I wrote a program for Budach's group that was supposed to take us scientifically forward to high level oriented on leading mathematicians in the world, mainly to Moskau, Paris, USA.

Politically, the times (1967/68) were also explosive in the East of Europe. I remember only the events in Czechoslovakia. There were big student demonstrations in Paris and West Berlin. Field medalist Alexander Grothendieck, whose groundbreaking work we studied, dedicated himself with his appeal "Survivre" to worldwide disarmament. We turned to him for inviting him to a lecture at Humboldt University. His answer was affirmative. He'd like to come, but he'd also like moreover to discuss with the students in public. But then there was a cold shower from the party head of the mathematics section: "This pacifist shouldn't come into our house".

In the run-up to the Prague Spring I also came across the following opinion of some students at the university: "The two systems - capitalism and socialism - are moving towards a similar society". This view ran under the name "Convergence Theory". That also sounded for us a bit mathematical, even more so with the abbreviation "K-Theory". Because in the seminar we also dealt with the mathematical K-Theory, with the help of which the Riemann-Roch

theory of higher dimensions could be better understood. Now I also had to deliver a socio-scientific paper to complete my doctorate. "Section of Social Sciences gave me the topic of "Social Convergence Theory", with the aim of disproving it. I asked for literary help and until the completion of my work (approx. three months), I was given the opportunity to inspect the "poison room" of the Staatsbibliothek Unter den Linden. I immediately noticed the collection of Nazi newspapers behind the door: "Der Völkische Beobachter" and "Der Stürmer". They couldn't help me, of course. I also discovered the "SPIEGEL", which I immediately learned to appreciate. Furthermore the following authors were a.o. on the GDR-index: Ota Šik (The red Ehrhard from Prag), Karl Jaspers, Z.Brzińsky, Kenneth Galbraith, Jan Tinbergen (Nobel prize winner in economy). They (and many others) were now accessible to me. I succeeded in convincingly arguing against the convergence thesis. It was a Cold War construct from the west side. The further development until the collapse of the Soviet Union should prove me right.[6] Anyway, I put the topic of K-theory on the agenda of one of our autumn schools. We had a lively two-hour exchange of opinions.

After I became an assistant at the Section of Mathematics in 1969, I suddenly had little time. In addition to seminar group leader and two practice groups, I was also assigned the leadership of a research seminar with the topic "Algebraic Surfaces" after the book of the same name - fresh translation of a brochure of the Steklov Institute Moscow with editor I.R.Shafarevich. It wasn't easy to understand, but pretty valuable for my future. I was also delegated to a course in Leipzig to learn programmed teaching. This was a low-result track on which I was pushed, as was Category Theory, too. I only had to drive to Leningrad for a year to complete my postgraduate studies. There I started a race to catch up with a new interesting topic of Algebraic Surface Theory of internationally interesting level.

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[6]Today, one would have to rethink this in view of the developments in China and the USA, for example.

Figure 13: M.Rapoport (∗1948 Cincinatti, USA), R-P.Holzapfel

8 Michael Rapoport

The Parents

His mother (Ingeborg Rapoport) was born in 1912, grew up in Hamburg, studied medicine there and graduated in 1937 with the state examination. However, she was not allowed to defend her dissertation because of the anti-Jewish legislation at that time, which, however, she made up in 2015 with bravura at the age of 102 in Hamburg. I.Rapoport emigrated to the USA in 1938, met her future husband Samuel Mitja Rapoport in 1944 at the University of Cincinnati. He had a littlebit earlier emigrated from Austria[7]. After the birth of their second son (Michael Rapoport) they had to leave the country again in order not to be accused in the McCarthy era for "un-American behavior". Thus they arrived via Austria in the GDR, where they both had a remarkable medical or biochemical-scientific career[8].

Michael Rapoport

Because of the origin of his father he possessed the Austrian nationality, and he did not give it up when he reached the age of majority in the GDR. He once jokingly remarked that he could still become president of the USA because of his country of birth. M.Rapoport received his school education in the GDR. At the (mathematically oriented) Heinrich-Hertz-Gymnasium in Berlin-Adlershof he graduated from high school in 1967. He then began to study mathematics at the Humboldt-University Berlin with strong support in the Budach Learning and Research Group.

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[7]https://de.wikipedia.org/wiki/Ingeborg_Rapoport
[8]https://de.wikipedia.org/wiki/Samuel_Mitja_Rapoport

After his diploma he continued his studies in Paris, Princeton and at the Harvard University (Cambridge, Massachusetts). The Fields medalists P.Deligne (Paris) and D.Mumford (Harvard) further encouraged him to reach world class level. He received his doctorate in 1976 from the University of Paris-Sud under Pierre Deligne with the thesis theme "Compactifications de L'espace de Modules de Hilbert-Blumenthal".

Then he returned to Berlin, where he worked as an assistant in the Section of Mathematics until 1980. Unfortunately, M. Rapoport was not employed according to his high level of scientific knowledge. The political component desired at the time was lacking. H. Koch wanted to get him into his research group at the academy with the consent of all employees of the group members. But there was no agreement either from the institute management or from Rapoport himself.

He left the GDR for the Federal Republic of Germany. From 1982 he successfully held professorships in Heidelberg, Bonn, Wuppertal, Köln, and since 2003 he held the chair of Arithmetic Algebraic Geometry at the University of Bonn. The numerous awards are listed in the Wikipedia article. His top performance is now the introduction of Peter Scholze, after his Abitur at the Heinrich-Hertz-Gymnasium, to the Fields Medal 2018. "I'm bursting with pride", was his reaction shortly after the award ceremony. This can be felt without restrictions.

9   The Heinrich-Hertz-Gymnasium

In 1961, the Extended Secondary School (EOS) located in Berlin-Adlershof was named after the physicist Heinrich Hertz. The profiling in the direction of mathematics only developed gradually with the lively support of the HU professors Hans Reichardt and Heinrich Grell. The first mathematically oriented 9th grade was established in 1963. From 1965 the Heinrich-Hertz-Oberschule became a school with a mathematical emphasis on. In 1969 all classes of the school were mathematically oriented special classes. Among pupils interested in mathematics and the natural sciences, more such schools were arranged, in almost all 16 GDR-districts, see Figure 14. It happened similarly as in the GDR sport.

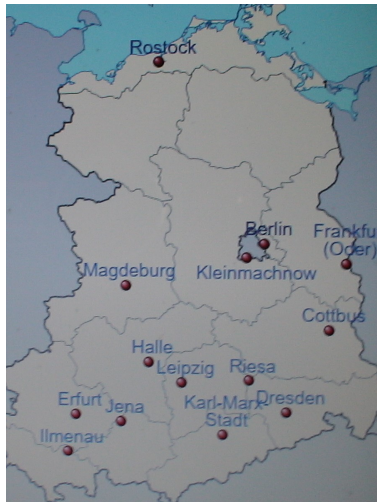A contemporary witness should now speak about the atmosphere

Figure 14: Math Special Schools in GDR

at the school in the 1960s, namely Ingeborg Rapoport[9]:

"This is the only school I hated in my life with all my heart, although it was technically excellent and, in fact, gave the pupils a very solid preparation for mthematics, natural sciences, but also for humanities subjects.

In contrast to the special classes at the Humboldt University, in which with great sensitivity to the political questions and doubts, the young people's own search for individual answers was addressed, a relentless sectarian spirit prevailed at the Heinrich Hertz School at that time. The adolescent children, all little 'beasts of intelligence', of course had many questions of a political nature, ideological abdominal pain and, of course, a need for personal judgment. There were many passionate discussions between my sons and (my husband) Mitja during her puberty, which were often led too hard by their father. ..... At home such discussions were played out, as painful as they were sometimes, on an underground of strong and unshakeable love. At the Heinrich-Hertz-Gymnasium under its director there was deep mistrust against these children at that time. Each of their questions was interpreted as a political provocation, and I don't doubt it either, that from the originally innocent questions and remarks a rebellious tone developed gradually - an evil

[9]I.Rapoport, Meine ersten drei Leben, NORA-Verlag, 1997, S.330 succeeding

Figure 15: Michael Rapoport, Thomas Zink

Circulus Vitiosus, which finally closed up in such a way that six children of the class as 'hostile elements' threatened to be expelled from school, ncluding Michael. There were, as far as we parents could see, only the German (femal) teacher who tried to find a reasonable counterweight against the true witch-hunt atmosphere at school.

I don't want to completely condemn Michael's time at the Heinrich Hertz School in the ground. It also had a decisive positive factor for his more distant future. A young professor of mathematics at Humboldt University, later member of the Academy of Sciences, Professor Budach, was interested in the gifted children and offered to work with them on his research topic. At first there were five or six who wanted to participate, but in the end there were only two: Michael, and his lifelong friend Thomas Zink[10].

With the turning point in 1990, all GDR special schools had to reorient. 1991 The application for the establishment of a school with a mathematical was approved. There is currently an advanced course for mathematically gifted puples, that already deals with study-relevant material including, in the case of very good performance, the receipt of valid university performance certificates. Advanced courses in mathematics and physics are available for all pupils, e.g. Introduction to the "Theory of Ordinary Differential Equations" or to "Metric Spaces".

Successful graduates of the Heinrich-Hertz-Gymnasium:

1966: Gregor Gysi, Politician;
1967: Michael Rapoport, Mathematician;
1967: Thomas Zink, Mathematician
1970: Klaus Altmann, Mathematician;

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[10]There were three: Gerd Siebert has been forgotten.

1976: Jürgen Kuttner, Radio Presenter;
1983: Yuri Tschinkel, Mathematician;
1985: Alexander Schmidt, Mathematician;
1985: Daniel Huybrechts, Mathematician;
2007: Peter Scholze, Mathematician, Fields Medal Winner 2018

Critical comments on the (German Wikipedia) school chronicle:

- Not mentioned in the school chronicle of the HHG was Thomas Zink.

- It should also be noted that the main initiators of mathematics profiling in the 1960s are not mentioned in the school chronicle. Abroad, especially in Great Britain, on the other hand, the merits of H. Reichardt and H. Grell are appreciated. The following text by the authors J.J.O'Connor and E.F. Robertson can be found on the Internet[11].

  "We should mention another aspect of Reichardt's contributions, namely his work with the Heinrich-Hertz-Gymnasium in Berlin. ... The school only received its mathematical emphasis in 1965 through the efforts of Reichardt and his colleague at the Humboldt University in Berlin, Heinrich Grell"[12].

Only once I attended the HH-Gymnasium myself. I'll come back to that in Section ref"Faltings".

10 Founding Director F. Hirzebruch of the Max Planck Institute for Mathematics

The life's work of F. Hirzebruch shall not be described here[13]. Rather, I present a few episodes of an East-West odyssey, which enabled me to make a professional (and human) approach. His name first struck me (1972 in Berlin) as the author of the Russian monograph "Topologitscheskije Metodi w algebraicheskoj geometrii".
The original was published (in English) by Sprimger already 1966. The translation came from B.B.Venkov. We could buy the Russian version in the GDR (for 5 DDR marks) and read it.

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[11]www-history.mcs.st-and.ac.uk/ Biographies/Reichardt.html
[12]Deutschsprachiger Bezug: $https://de.wikipedia.org/wiki/Hans\_Reichardt$
[13]Hierzu sei auf $https://de.wikipedia.org/wiki/Friedrich\_Hirzebruch$ verwiesen

In September 1973 I was delegated to the Soviet Union for one year of additional studies. Like almost all delegates, I had stated Moscow as my desired destination, but was not heard. There, at the Lomonossow University, one had best working conditions (with own room) and good scientific care. I wanted to make my outward journey individual, because I wanted to enroll my daughter in school at the beginning of September. That was not approved; I had to go with the common delegation train. In Moscow we were then sent in smaller groups on trains to the respective destinations departing from different stations. I was promoted to Leningrad (today St. Petersburg), there to the student dorm on the Vasilyev Island, Detskaya Uliza 10. I moved into a room together with two Russian students and a colleague from Dresden. We shared kitchen, laundry room and toilet with the students of the entire corridore.

The next day, we new arrivals registered us at Leningrad University, where we should be divided among our caregivers corresponding to our levels, which have been checked. In the mathematics room sat none other than Z.I.Borevich (1922-1995), whose book "Number Theory", which he had written together with I.R.Shafarevich, was well known among algebraists. We spoke about international works that had already been studied at home, including Russian ones (Manin, Shafarevich). He said: "I'm sorry, there's no way you can learn more from our chair holders here". I thought to myself: So I am obviously sent into a desert by the GDR's Ministry of Higher Education (or by a group of administrators behind).

Since we also knew some GDR scientists together from the research groups of H.Reichardt and H.Koch, Borevich said I didn't have to come here for nothing. Not far from here, at the Fontanka, there is the Leningrad department of the Steklov-Institute for Mathematics of the Academy of Sciences. There works B.B.Venkov, from which you can still learn a lot. Aha, the translator of the Hirzebruch book. I contacted him, and he invited me to his research seminar on Algebraic Geometry and Number theory. The current work of the Italian Fields Medalist E.Bombieri has just been dealt with.

After three weeks I noticed that I could have stayed at home for it. I wanted to directly participate in a current Russian research project. "All right", he said. "I have a Moscow dissertation to review, let's inspire you, come on board with me". That's what I wanted to do. This initially involved some time with the transla-

Figure 16: Friedrich Hirzebruch, 1927-2012

tion from Russian into German, in writing and formally, with little understanding the mathematical context. For this I lacked various classical and modern knowledges yet. But the combination of differential geometry, number theory, topology, analysis and algebraic geometry fascinated me immediately. At home, in the GDR the directions ran in separate research groups, whose leaders scarcely communicated with each other. A race to catch up had to begin in order to understand everything and and then to open up further surface constructions, beyond the Moscow doctoral thesis.

B.B.Venkov now referred me to works by Hirzebruch, in which he had immersed himself and kept a constant eye on mathematical news from Bonn. He noticed that I could understand important works from there particularly well linguistically. "That's where the future lies", he said. Before my return to Berlin I asked him to give me this sentence in writing, n order to be able to continue working in this direction at Humboldt-University. I presented Venkov's recommendation to my section at home, but there reaped incomprehension. A colleague patted me on the shoulder a little later and told me about a party meeting, where it was discussed that we do not want to accept issues from NATO countries. On the other hand

one could read on banners: "Learning from the Soviet Union means learning to win." A contradiction, which I had to solve for myself. I said to me with a title from the Beatles in my head: "I'll follow the sun".

The Hilbert modular surfaces, that were processed by Hirzebruch and some of his students, now also belonged to the race to catch up. They offered a nice guideline. The surfacess that came into view with the Moscow dissertation, I called "Picard Modular Surfaces", because they jumped out of E.Picard's work a hundred years earlier for monodromy investigations (ambiguities) of special complex functions in two variables. The modularity could also be explained. For some more details see Section 11.

Unfortunately, I had to interrupt my work because of a call up for six-month reserve service in the army (NVA, Nationale Volksarmee) as "Soldier Dr." Holzapfel. I kept reading Hirzebruch's articles in my rare spare time. After all, a whole company now knew, that science in Bonn was much further along than we were in the GDR. All the comrades found this interesting. That's what the state security also called into question. One day, there was a special inspection of my wardrobe. They fished the Hirzebruch works out of one compartment. After a week, I got it all back with the remark that they didn't found any political text. I was allowed to read on.

At home, my doctoral student J. M. Feustel - son of the "Pittiplatsch" creators of GDR television - continued to work diligently on the Picard modular surfaces. After a very good diploma thesis on the same topic he got a job in the Koch research group at the Institute for Mathematics of the GDR-Academy of Sciences, without any teaching commitments.

After my army service, the two of us went to work together. It must have been 1977 when a visit of Hirzebruch in East-Berlin together with some other BRD-mathematicians was announced. At our section, counselors for the guests have been assigned, who will pick them up at the border station Friedrichstrae to escorte them to their hotel. The correspondences were not professionally appropriately distributed, more or less by random. With a colleague I agreed on a swap: I was drawn to F. Hirzebruch. On the way to the hotel I immediately got into mathematical conversation with Prof. Hirzebruch. I've been able to show some results. Hirzebruch was immediately interested; he quickly found access to my train

Figure 17: Reinhardsbrunn Palace 2017

of thought. After the lectures of our guests we sat together even longer. He gave me valuable advices and also gave me the latest work from his Ph.D. students who worked on Hilbert modular surfaces. In the course of one year I wrote the first two chapters of my "Ball Quotient Surfaces" as a brochure in the "Seminar Reports" of the Sektion Mathematics of the Humboldt-University. I sent one copy to Bonn.

In one May week 1978 the Leopoldina Symposium "Singularities" was organized by the MPI (Hirzebruch) in Reinhardsbrunn Palace (Thuringia). The Leopoldina, founded in 1652 is the world's oldest permanent nature research academy. Since 1878, its headquarters have been located in Halle (Saale, East Germany). As the oldest scientific-medical scholarly society in the German-speaking space, it remained largely free of state influence and remained true to its all-German character, even in the GDR years. It should also be mentioned that the Academy was appointed "National Academy of Sciences" in July 2008. More historical details can be found on[14]. Today it seems to me as if the annual "Arbeitstagung" of the MPI in Bonn had been moved to the GDR along the Leopoldina line. For example, the well-known international participation from East and West was of this level. I particularly remember the participation of the Fields Medalist David Mumford (USA) as well as the singularity specialists E. Looijenga (Holland), H. Pinkham (USA) and E. Brieskorn (Bonn, master follower of Hirzebruch). The latter insulted very much that all invited Russians were denied leave of the Soviet Union for the GDR. Brieskorn would have especially liked to meet W. I. Arnold (Moscow).

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[14]wikipedia.org/wiki/$Deutsche_-Akademie_-der_-Naturforscher_-Leopoldina\#Geschichte$
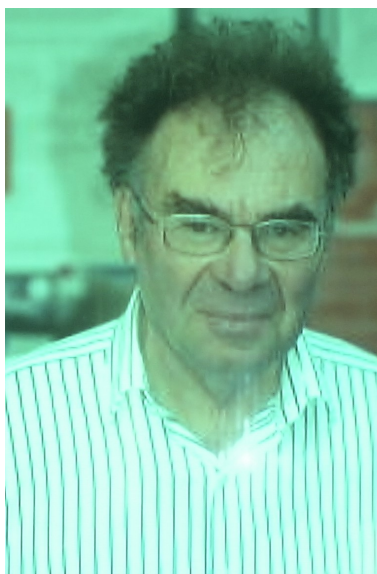
Figure 18: Günter Harder, *1939

Also the accommodation in the beautiful castle Reinhardsbrunn (near Friedrichroda) seemed to me to represent somehow an all-German line. I was astonished that the Bible was to be found in every bedside table. In fact, since 1953 the castle had served as an (exquisite) hotel for the "VEB Travel Agency" of the GDR, above all as a source of foreign exchange for guests from non-socialist countries. It also developed into a cultural and educational centre, where concerts and congresses took place. To the history of many years (1085 - 2018) we refer to[15]. Last year (2018) it was (once again) expropriated, this time in accordance with the Monument Protection Act.

Now it was also established that no GDR mathematician was nominated for a lecture at all. E. Brieskorn took the floor to suggest that he would like to hear a contribution from Mr. Holzapfel. I myself noticed that I was well prepared for giving a lecture. The proposal was adopted. The lecture was - as I learned later - well judged from different sides (also from USA). My preprints were all taken.

After the fall of communism, the Academy of Sciences of the GDR

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[15]https://de.wikipedia.org/wiki/Reinhardsbrunn

Figure 19: Juri I. Manin, *1937

was wound up. Our research group "Number Theory and Algebraic Geometry" was evaluated by G.Harder of the Max Planck Institute for Mathematics in Bonn. We - H. Koch, E. -W. Zink and me - were declared as the "Max Planck Group at the Humboldt University in Berlin" for a period of (maximum) five years. Until we were awarded a chair, however, we were funded directly from the MPI in Bonn. A meeting with F. Hirzebruch (1991) in a villa in Berlin-Grunewald suggests that we owe this path to him.

By the way, G. Harder was also director of the Max Planck Institute from 1995 to 2006 (after the retirement of Hirzebruch).

11   Juri Iwanowich Manin (*1937)

J. I. Manin (Moscow) was appointed Co-Director of the MPI in 1995. He stayed it for ten years. I knew him professionally from his book "Kubicheskije Formi" and and his proof of the "Mordell Conjecture" for function fields with the help of the Gauß-Manin connection.

Personally I got to know him in a children's holiday camp of the GDR. There a well attended International (East-West) Conference took place. For cost reasons, a children's holiday camp in Garwitz (near Parchim, Mecklenburg) was resuscitated together with staff outside the holiday season[16]. Would also be a good idea today for

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[16]www-irm.mathematik.hu-berlin.de/ pahlisch/Garwitz/garwitz.html

clammy locations for better integration into the economic cycle.

I was introduced to Manin during a paper I submitted for the newly founded "Annals of Global Analysis and Geometry". He praised the writing style and the high proportion of arithmetic in the geometric theme. Also, that I will attack the continuation of a Moscow dissertation, he found sympathetic. He asked me to accompany him - before his flight home to Moscow - on a short visit of Berlin. At his request we visited the Pergamon Museum. There were interesting conversations about mathematics and the city. He was interested, for example, in how I came up with the name "Picardian Modular Surfaces" of my research objects. The monodromy groups Γ of special differential equation (solutions) operating on the 2-dimensional complex unit Ball $\mathbb{B} = \{(z_1, z_2) \in \mathbb{C}^2; |z_1|^2 + |z_2|^2 < 1\}$ were discovered by Émile Picard in 1883. The base area of the solving functions can be interpreted as the module surface of an associated curve family.

He was also interested in M. Rapoport, whose works in Moscow were known. Why can he - coming from the GDR - work in Western Europe and America, while we're not allowed to accept such invitations ? The matter was - referring to Rapoport's Austria Pass - quickly explained.

In the meantime (1979) I had left my employer (Humboldt-University) after being invited by Professor H.Koch to join his research group at the Academy in Berlin-Adlershof. Now I could also participate in the academy exchange program with the USSR. My first (quarterly) stay took me to Moscow, where I was immediately invited for giving a lecture at the Vinberg Seminar at Lomonosov University. I met J. Manin again at the Steklov Institute. He invited me to his house, which I felt was a special honor for me. Again there was valuable mathematical advices for my book on "Euler Partial Differential Equations" (EPD) on which I was working. I remember a conversation about the Soviet writer and Nobel laureate Boris Pasternak (author of "Dr.Shivago"). Pasternak is mentioned by name at the beginning of one of Manin's books. I asked: "But why is he being fought by the Soviet authorities" ? "There's no understandable reason", he replied, "Think of Heinrich Heine in your country". It was an analogy I could well understand.

Due to Manin hints, e. g. on Delignes Hyperkohomologie groups/ sheaves, I was able to give the first Euler lecture in Germany. This
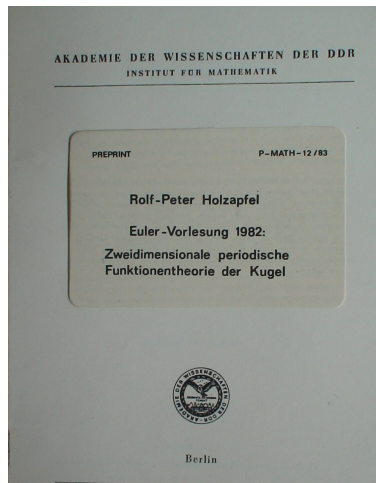
Figure 20: First Euler Lecture, 1982

happened at the Central Institute for Mathematics and Mechanics of the GDR-Academy. Moreover, it was possible to complete the writing of my book EPD, which appeared as well in GDR as in Holland (Reidel Publishing Company, East European Series). I was then (1985) awarded with the Euler Medal of the GDR.

12   Gerd Faltings (∗1954)

Gerd Faltings was the first German mathematician to win the Fields Medal, namely in 1986 for the three years before successful proof of the Mordell presumption. It is included (for $K = \mathbb{Q}$) in the following

Theorem of Faltings. Each (complex compact smooth) curve of genus $g > 1$, defined over a number field K, has (at most) finitely many K-points.

I knew G. Faltings personally from conferences in the early 1980s. In Romania we went for a walk together on the day of the trip. He inquired about my work and that of some Moscow colleagues. Especially Manin, Shafarevich and Parshin approached the proof of the 60 years old assumption of Mordell. I noticed right away that Faltings was a highly talented Mathematician. In the run-up to the Algebraic Geometry Conference in Bucharest. In 1983 there

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Figure 21: Gerd Faltings *1954, Fields-Medal 1986

was already rumour about the remarkable result of Faltings. His lecture was then, so to speak, the foreign premiere of the proof of the Mordell conjecture.

Personally he gave me his 40-page preprint with the proof. I didn't take good care of it, and it was stolen from me. Then I had to order it again from Berlin. Back to the Bucharest conference: I have them in good memory, both mathematically and in human respects. We sang together cheerfully in the evening at the Conference Party, e. g. "We shall overcome". I tuned in to the song of "It's a long way to Tipparary" the mathematical text invention: "It's a long way to blow down threefolds, it's a long way but we blow". Several such mathematical contributions were now sung along. "Long way to prove conjectures" was also not a bad text.

After being awarded the Fields Medal at the ICM in 1986, my former colleague Reinhard Blling invited me to the Heinrich Hertz School. There Blling led the special course for the Gymnasium students. Of course my topics included: Mordell Conjecture, the Fields Medal and my acquaintance with G. Faltings. It was the only time I had direct contact with the Heinrich-Hertz-Gymnasium.

Faltings went to Princeton as a professor in 1985-1994. The following year he returned to Germany. Since then, he has been co-director of the Max Planck Institute for Mathematics in Bonn.

Figure 22: Peter Scholze ∗1987, Fields-Medail 2018

13 Peter Scholze (∗1987)

13.1 Wissenschaftliche Biographie

He was born in 1987 in Dresden. He grew up in Berlin and attended the Heinrich-Hertz-Gymnasium at his new place in Berlin-Friedrichshain. From the age of sixteen he was sponsored by Professor Klaus Altmann at Freie Universitt Berlin. After a short time he also took part in mathematics seminars at the university. In an e-mail he wrote:

"My time at the Heinrich Hertz School was formative for me, and has contributed greatly to my development. The influence of the school extends over my entire mathematical career, from the first beginnings at the Mathematics Olympiad, which I would not have mastered so successfully without the preparation in the AGs at school, about the first lectures at the university, which I was already able to visit during my school time with Professor Klaus Altmann, a former Heinrich Hertz pupil."

Peter Scholze finished the Heinrich-Hertz-Gymnasium with a grade of 1.0 in 2007.

In mathematics lessons Peter Scholze always worked in "parallel". On the one hand, he followed the teaching process with "half an ear". He intervened in the discussion if he didn't like something right or if the solution to the problem was too long in coming. He then shook up suggestions for solutions and was able to present them - mostly smiling - cleanly and comprehensibly for everyone on the blackboard. At the same time, P.Scholze was always immersed in mathematical literature during his lessons or solved higher-level Mathematical Olympiad tasks. In the afternoon he then visited

mathematics working groups at the school or events of the Mathematical Puples Society "Leonhard Euler" at Humboldt-Universitt.

In the 11th, 12th and 13th grades we as Heinrich-Hertz-Schule could no longer offer "our Peter" too much mathematically. Then our former student, Prof. Dr. Klaus Altmann (graduate of our school in 1975) came to our aid. P. Scholze was individually supervised by Klaus Altmann in the 11th to 13th grade at Freie Universitt Berlin and attended our distinguished mathematics performance course only sporadically. During this time Peter began to work seriously and systematically in the field of algebraic geometry.

At the age of 16 years he learned that ten years earlier Andrew Wiles had solved the famous 17th century Fermat problem: Prove that the equation $x^n + y^n = z^n$ has no solution in natural numbers if the exponent n is greater than 2. Scholze wanted to understand the evidence, but soon had to realize that the task was easy to formulate, that while the task is simple to formulate, but the solution requires some sorts of the hardest current mathematics. Scholze noted: "I didn't understand anything at first, but it was fascinating."

The student then worked his way through the material from back to front: Every time he did not understand something, he appropriated the necessary material, then what he needed to understand that content, and so on. "That's about how I'm still learning today", he explained in a conversation. "What you do in the beginner lecture, like linear algebra, I never really studied - I've noticed that on other occasions".

From the age of eighteen he was promoted by Michael Rapoport - Professor for arithmetic algebraic geometry at the Rheinish Friedrich-Wilhelms-University Bonn. Within five semesters, he completed his studies in Bonn with the master's degree. In 2012 Scholze was awarded his dissertation under the title "Perfectoid Spaces". In the winter semester 2012/2013 he was appointed to one of the Hausdorff Mathematics Professorships at the Excellence Cluster in Bonn. His area of expertise is Arithmetic-Algebraic Geometry with the Langlands program as a special framework. 2018 he became one of the directors of the Max Planck Institute for Mathematics.

13.2 On the way to the Fields Medal 2018

His most significant concept up to date is a class of fractal structures, which he called "Perfectoid Spaces". The idea is only a few years old, but has already borne rich fruit in algebraic geometry. The discovered spaces are indeed very large and complex, but have useful geometric properties for many problems. With their help a whole series of old difficult problems of number theory could be solved in the meantime.

In the 1970s, mathematicians noticed that p-adic numbers were easier to come by, when you're not just studying them by oneselves, but builds on them an infinite tower of number systems: At the bottom the p-adic numbers themselves, above a system of p copies of the p-adic numbers arranged in a circle, and so on. Each floor is in a sense a p-fold version of the one immediately under it. In other words, perfectoid spaces are "infinite towers" of ever larger number systems. The main role play the p-adic numbers. "Meanwhile I am so used to them that the real numbers seem strange to me", Scholze told us in an interview.

In 2010, there was a rumor among number theorists. Allegedly, a student from Bonn had written a paper taking only 37 pages, for which the two established scientists Michael Harris and Richard Taylor had needed a book of 288 pages: The impenetrable evidence of a proposition from number theory, which in turn is about a special case of the famous Langlands Correspondence. Indeed, In his master thesis, the 22 years old Peter Scholze replaced one of the most complicated parts of the proof with an elegant construction of his own.

In the middle of the 20th century the mathematicians (in particular Robert Langlands) discovered[17] a relationship of reciprocity laws to an - apparently - completely different subject: the hyperbolic geometry.

The connection between the reciprocity laws and hyperbolic geometry is a core element of the above mentioned Langlands Program: A Collection of propositions an conjectures, which join number theory, analysis and geometry with each other. Once proven, the assumptions allow far-reaching conclusions to be drawn. Thus it is possible to trace the proof of Fermat's Last Theorem back to a small

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[17]Winner of Abel Prize 2018

- but far from being simple - part of the Langlands Programme.

In the overview article "p-adic Geometry", which P. Scholze wrote for the International Mathematicians' Congress 2018 in Rio de Janeiro, he not only summarizes his research on the strange and rich world of p-adic structures. He also describes their relations to the famous Langlands correspondence.

Hereby we were - even if we understand much only rudimentary - witnesses of a scientific volcanic eruption, whose smoldering began 120 years earlier with the invention of the p-adic numbers by Kurt Hensel in Berlin.

13.3   Press comments in August 2018

In an article of its last July issue 2018, P. Scholze was named "Mozart of Mathematics" by the SPIEGEL. He is introduced as a mathematics researcher of the most remote spheres of number theory. His colleagues praise him as friendly, down-to-earth and always approachable. A mathematician like him only exists once every few decades, says his doctoral supervisor M. Rapoport, himself a grandmaster of his trade. Scholze is on the move at the level of abstraction, in the other colleagues the air becomes too thin, in a death zone, so to speak. As a student of 22 Scholze tackled an assumption known as local Langlands correspondence. It is about the transformation of number systems into geometric entities, which then allow an easier access.

Already during his time at the Heinrich-Hertz-Gymnasium he read difficult textbooks on the side. He followed the lessons only with half an ear, so his teachers there report. Even then he never wrote notes. To this day Scholze does almost everything in his head. If he ever forces himself to take notes, he won't notice anything, he explained. In this respect, he is compared to Mozart, who once composed an opera entirely in his head and wrote it down afterwards. Henri Poincaré also occurs to me (the author) in this context, who could completely reconstruct a lecture only by listening, without - due to a visual impairment - recognizing the panel painting.

On the title page of the BERLINER KURIER on August 2 (2018), the line appears under the heading "Berlin's brightest head, Genius East": Peter Scholze (30) wins the world's most important award for mathematicians, and he says: "I owe this also to the

school system of the GDR". On the text page you will find the corked header line: Top prize for calculating genius from Berlin. And further in journalistic pun style: There's more under his mat than math.

In a letter to his former Heinrich Hertz school he writes: "I hope that future generations will also be lucky enough to be taught in a network of mathematically and scientifically outstanding schools."

Also DIE WELT from August 2nd printed his portrait on the title page with reference to an interview on page 28. There he tells: "All my strength flows into mathematics". He has the charm of a young student and yet is one of the most brilliant mathematicians of our time. In the children's book "Fregattenkapitn Eins" he found his first contact with mathematics. Scholze goes on to say: "It makes no sense to simply want to attack these still unsolved problems. They're just too difficult for that. Only if you've come to a new conclusion anyway, that might be a key to solving one of these problems, there's a chance. Only then would it make sense to try it."

Scholze also knows the feeling of failure when, after weeks of work, no result has been achieved. "It's even worse when you think you've found a proof, then after a month you have to find out that it doesn't work. That's frustrating."

"To me, it feels like I'm discovering mathematics. In a way, my approach is a natural scientific one. I try to understand structures that already exist objectively. I don't have a choice how to attack things. The thing dictates what I have to do."

"My role model is the mathematician Alexander Grothendieck, who died in 2014. He has put algebraic geometry on a new footing and created the prerequisites for my work."

Finally, Erica Klarreich: "At just 30 years of age, Peter Scholze has turned a whole field of expertise upside down. In return, he rightly receives the Fields Medal."

13.4   Echo of some international colleagues

Jared Weinstein, Number theorist at Boston University:

It was unbelievable that someone so young could do something so revolutionary. He can see the developments before they have even begun; he has foreseeing qualities. The craziest thing about the perfectoid spaces is that you can miraculously switch from one number

system to another with them. In 1970 Pierre Deligne formulated his 'weight-monodromy conjecture'. With the proof, P. Scholze received his doctorate in 2012. His dissertation had such far-reaching consequences that working groups all over the world became absorbed in it. 'Perfectoid Spaces' are undoubtedly one of the most difficult of all in mathematics. Nevertheless, Scholze is known for the clarity of his representations, both in articles and in lectures. I don't understand anything until Peter explains it to me. While his colleagues all over the world are struggling with the 'Perfectoid Spaces', the most far-reaching discoveries in this field, not surprisingly, come from Scholze himself and his colleagues at work. A result that he posted on the Internet in 2013 caused incredulous amazement among experts. We never thought that such a proposition was even within reach. In this work Scholze succeeded in generalising the so-called 'Reciprocity Theorems'. It deals with polynomial equations in modular arithmetic. One can interpret larger parts of modern algebraic number theory as the attempt to generalize these laws.

Bhargav Bhatt, University of Michigan:

Scholze is still working on the 'Perfektoid Spaces'. But now he's taken a trip to algebraic topology. That is the discipline which examines deformable structures by means of algebra. In just one and a half years he has become an absolute expert in this field. The colleagues in this direction see it with a mixture of fear and enthusiasm when Scholze enters her kingdom. This means that a lot will happen in the near future. One will be able to observe the limits of knowledge while hiking.

Eugen Hellmann, former fellow student:

It is due to his phenomenal ability to look deeply into the essence of a mathematical object. In contrast to many of his colleagues, he does not seek the solution to a specific problem, but a misterious concept, that he wants to understand for his own sake. P. Scholze never wrote anything down. He understood that right away, and not just in any way, but so deeply penetrated that he never forgot.

Scholze has found the clearest form in which one could express all previous results, and this in an elegant way. Since that was exactly the right concept, he could go far beyond the known things. Even with an introduction by Scholze, 'Perfect Spaces' are anything but

easy to understand. If you deviate even a little bit from the path he gives you, you'll end up in the darkest jungle, and then it gets really difficult. But Peter never stuck in the jungle. He's not even trying to smash through. He's flying over it. It was like a flight over the previous jungle.

Ana Caraiani, Numbers Theory, at Imperial College London:

Afterwards, the structures that he has devised found millions of applications, that no one could have imagined before, precisely because it had exactly the right concepts were. Scholze is not too sorry to be able to put his ideas on an intermediate diploma level for explaining. Thanks to his friendly, accessible nature, he is the ideal leader in his field.