Warm Up

Differential Geometry WS 2019/20

Problem 1 Differentiable maps and implicit function theorem

Let $f : U \subset \mathbb{R}^m \to \mathbb{R}^n$ be a function defined on an open set U.

a) Explain what it means that f is of class C^1, i.e. continuously differentiable.

b) Describe the "first derivative" and explaion its meaning.

c) Let $g : \mathbb{R}^n \to \mathbb{R}^p$ be another C^1-function. Formulate the chain rule for $g \circ f$.

d) Formulate the inverse function theorem and the implicit funtion theorem.

e) For f as above and $x \in U$ assume that the differential of f at x is surjective. Describe $f^{-1}(f(x))$ near x as a graph of a differentiable function.

f) Study the different situations for $f : \mathbb{R}^n \to \mathbb{R}$, $f(x_1, ..., x_n) = x_1^2 + ... + x_n^2$ for $n = 2$ and $n = 3$.

Problem 2 Curves, length of a curve, first and second derivative

Let $\gamma : I \to \mathbb{R}^n$ be a C^1–map, $I \subset \mathbb{R}$ is an interval.

a) Interpret first and second derivative of γ in physical terms (classical mechanics). Formulate Newton's laws using differential calculus.

b) Define the length of γ. Explain why it does not depend on its parametrization.

c) Assume that the value of the velocity of a pointlike mass moving according to Newton's laws is constant. What can you conclude about the force acting on it?

d) Let $f : U \subset \mathbb{R}^m \to \mathbb{R}$ be a real C^1–function ($U \subset \mathbb{R}^m$ again open). Let $\gamma : I \to U$ be C^1 such that $f \circ \gamma$ is constant. Show that $\dot{\gamma}$ is perpendicular to the gradient ∇f of f.