Problem Set 7

Differential Geometry
WS 2019/20

Problems 1 to 4 can be discussed in class.
You may submit solutions for Problem 5 until December 11.

Problem 1
(i) Describe the first fundamental of the sphere in the coordinates of Problem 2, Problem Set 6 (stereographic projection).
(ii) Use spherical angles (as for S^2) to define parametrizations of the ellipsoid

$$E = \{(x, y, z) \in \mathbb{R}^3 \mid \frac{x^2}{a^2} + \frac{x^2}{b^2} + \frac{x^2}{c^2} = 1\}$$

for $a, b, c > 0$ and describe the firts fundamental form in the corresponding coordinates.
(iii) Find suitable global coordinates for the double cone without the vertex,$C \setminus \{0\} \subset \mathbb{R}^3$, of Problem 4, Problem Set 6 and describe the first fundamental form in those coordinates.

Problem 2
Show Proposition 39: If $\Phi := \tilde{\varphi}^{-1} \circ \varphi$ is the transition map between the coordinates of two parametrizations $\varphi, \tilde{\varphi}$ of a surface $F \subset \mathbb{R}^3$ then the corresponding Gram matrices $g(.), \tilde{g}(.)$ for the first fundamental form transform as follows

$$g(x) = (d_x\Phi)^T \tilde{g}(\Phi(x)) d_x\Phi.$$

Problem 3
Which of the parametrizations of regular surfaces discussed in class or in the tutorial are angle preserving, i.e. conformal, i.e. if $\varphi : U \to F$ is a local parametrization, $p \in F$, $x \in U$ with $\varphi(x) = p$, $X, Y \in T_pF$ and $v, w \in \mathbb{R}^2$ such that $X = d_x\varphi(v), Y = d_x\varphi(w)$ we have

$$\measuredangle(X, Y) = \measuredangle_{\mathbb{R}^2}(v, w)?$$

Problem 4
Compute the first fundamental form of the graph of a differentiable function on two variables with respect to the obvious parametrization.

Problem 5
Describe the first fundamental form of $T^2 \subset \mathbb{R}^3$ if Problem 5, Problem Set 6 with respect to the following parametrization $\varphi : (0, 2\pi) \times (0, 2\pi) \to T^2$

$$\varphi(\alpha, \beta) = ((R - r\cos\beta)\cos\alpha, (R - r\cos\beta)\sin\alpha, r\sin\beta).$$