Problem Set 8

Differential Geometry
WS 2019/20

Problems 1 to 3 can be discussed in class.
You may submit solutions for Problems 4 and 5 until December 18.

Problem 1[Moebius strip]
(i) Define a regular surface in $\mathbb{R}^3$ which models the Moebius strip.
(ii) Show that the Moebius strip is not orientable using your model.

Problem 2
Let $F \subset \mathbb{R}^3$ be a regular, oirientable surface, $N : F \to S^2$ its unit normal field. Let $k \in \mathbb{N}, k \geq 2$. Show that if F is of class C^k then N is of class C^{k-1}.

Problem 3
Compute first and second fundamental form, as well as Gauss and mean curvature for the graph

$$G = \{(x, y, x^2 - y^2) \mid x, y \in \mathbb{R}\} \subset \mathbb{R}^3,$$

in the globally defined parametrization of G given by the graph. In addition compute the principal directions at $(0, 0, 0)$.

Problem 4
Regular simple or regular simple closed curves revisited:
Let $\gamma : I \to \mathbb{R}^2$ be a regular curve. Assume that $I = [a, b]$ and γ is injective or $I = \mathbb{R}$ and γ is periodic and a simple closed curve.
(i) Show that the image of γ (referred to as γ) admits a local parametrization by a differentiable map $\varphi : (-\epsilon, \epsilon) \to \gamma$ which is a homeomorphism onto its image. Find equivalent statements to that similar to regular surfaces (description as the level set of a differentiable function; description as a graph).
(ii) What should the first fundamental form look like (definition and description via local coordinates).
(iii) Do non-orientable regular curves exist? What is the Gauss map? What are corresponding notions for Weingarten map and second fundamental form? What is, in this context, curvature, what is its relation to curvature of curves defined before?

Problem 5
Let

$$F := \{(x, y, f(x, y)) \mid x, y \in U\} \subset \mathbb{R}^3$$

be the graph of a differentiable function (at least C^2) over an open subset $U \subset \mathbb{R}^2$. Compute its first and second fundamental form, its Weingarten map in the globally defined parametrization of F given by the graph. Derive formulas for its mean curvature and its Gaussian curvature.