

## BERKOVICH SPACES

College Seminar  
Summer 2016  
Wednesdays 13.15-15.00 in 1.023

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The main goal of the semester is to introduce ourselves to Berkovich spaces and some of their arithmetic applications, including Mordellic bounds on curves. We'll ease ourselves into nonarchimedean analysis by first over-viewing Tate's theory of rigid analytic spaces, and then move on to a detailed study of Berkovich spaces. Throughout we'll be looking to curves over valued fields and their analytifications as our primary source of examples, though we will discuss some higher dimensional geometry as the need arises.

I have as yet been able to find a good single source. There are some definitive but sometimes overwhelming references for rigid geometry, and usually the only available reference for Berkovich spaces are his papers. Bosch's book is probably the best introduction/reference for the general theory of rigid geometry and its connection to formal geometry, while Fresnel–van der Put is good for some in-depth geometric examples.

- (A) J. Tate. *Rigid analytic spaces*. Tate's original article. As with most of Tate's articles, very clear but inevitably outdated.
- (B) S. Bosch. *Lectures on formal and rigid geometry*. Very accessible and modern.
- (C) S. Bosch, U. Guntzer, and R. Remmert. *Non-archimedean analysis*. A standard reference, a bit less readable.
- (D) J. Fresnel and M. van der Put. *Rigid analytic geometry and its applications*. Extensively treats curves and abelian varieties.
- (E) V. Berkovich. *Non-archimedean analytic spaces*. Berkovich's Trieste notes. Good but formal.

Here are some notes available online that may provide some more useful descriptions:

- (A) B. Conrad. *Several approaches to non-archimedean geometry*. Carefully written notes on some foundational aspects of rigid spaces vs. Berkovich spaces.
- (B) M. Temkin. *Introduction to Berkovich analytic spaces*. More in depth notes on the same topic.
- (C) M. Baker. *Non-archimedean geometry*. Notes from a course given by Baker. Lots of good stuff here, and lots of examples. He has some other sources on his [webpage](#), for example [these notes](#) on the Berkovich projective line.

### LECTURE TOPICS AND NOTES

20.04.2016 Overview.  
27.04.2016 Tate algebra and affinoids.  
04.05.2016 Rigid spaces and analytification.  
11.05.2016 Uniformization of elliptic curves and the Tate curve.  
18.05.2016 Formal models and Raynaud's theorem.  
25.05.2016 Berkovich spaces I.  
01.06.2016 Berkovich spaces II.  
08.06.2016 —Kramer birthday conference—  
15.06.2016 Berkovich curves.  
22.06.2016 TBA.  
29.06.2016 TBA.

06.07.2016 TBA.  
13.07.2016 TBA.  
20.07.2016 TBA.