CPDE II, SuSe 19
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Exercise Sheet 2

Discussion on 07 May 2019

Please be prepared to present one or more of the following exercises on the blackboard. If not stated otherwise, algorithms or code can be displayed in Matlab or pseudocode.

Exercise 1 (cf. Tutorial on 23 April). Given two deformations $\varphi, \widetilde{\varphi} : \Omega \to \mathbb{R}^3$ with $\mathrm{D}\,\varphi^\top \mathrm{D}\,\varphi = \mathrm{D}\,\widetilde{\varphi}^\top \mathrm{D}\,\widetilde{\varphi}$ in Ω, show that there exists a rigid body motion $R \in RBM(\Omega)$ such that

$$\widetilde{\varphi} = R \circ \varphi.$$

Interpret this result.

Exercise 2 (Trivial Cauchy strain tensor). Prove that $v \in H^1(\Omega; \mathbb{R}^3)$ satisfies $\varepsilon(v) = 0$ if and only if there exist $a, b \in \mathbb{R}^3$ such that $v(x) = a \times x + b$ for almost every $x \in \Omega$.

Exercise 3 (Korn inequality). Prove the Korn inequality directly for the case of full Dirichlet boundary $\Gamma_\mathrm{D} = \partial\Omega$, i.e., for every $v \in H^1_0(\Omega; \mathbb{R}^3)$ it holds that

$$\| \mathrm{D}\, v \|_{L^2(\Omega)} \leq C_{\mathrm{Korn}} \| \varepsilon(v) \|_{L^2(\Omega)}.$$

Under which regularity assumptions on the boundary $\partial\Omega$ does this hold?

Hint: Use integration by parts and a density argument.

Exercise 4 (Poisson model problem in $\mathbb{R}^3$; implementation). Implement the solution of the Poisson model problem in $\mathbb{R}^3$ with mixed boundary conditions seeking $u \in H^1_\mathrm{D}(\Omega; \mathbb{R}^3)$ such that

$$\int_\Omega \mathrm{D}\, u : \mathrm{D}\, v \,\mathrm{d}x = \int_\Omega f \cdot v \,\mathrm{d}x - \int_{\Gamma_\mathrm{N}} g \cdot v \,\mathrm{d}a \quad \text{for} \quad v \in H^1_\mathrm{D}(\Omega; \mathbb{R}^3).$$

Hint: You can extend your favourite FEM software, e.g., the AFEM package from the lecture's homepage.

Literature. The following references concern the prerequisites from functional and numerical analysis required for this exercise sheet. Every reference is electronically available in the HU library and from HU intranet (e.g., eduroam).

- B. D. Reddy, Introductory Functional Analysis With Applications to Boundary Value Problems and Finite Elements, Springer, New York 1998.
  - Chapter I, Sections 7.3 and 7.5 for the density of C^∞ functions in H^1
- C. Carstensen, A young person's guide to the art of Finite Element programming, 2013.