CPDE II, SuSe 19
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Exercise Sheet 8

Discussion on 18 June 2019

Please be prepared to present one or more of the following exercises on the blackboard. If not stated otherwise, algorithms or code can be displayed in Matlab or pseudocode.

Exercise 1 (A posteriori estimates for LSFEM). *(a)* Let $\sigma \in \Sigma_{\mathrm{N}}$ and $u \in U_{\mathrm{D}}$ minimize the least-squares functional $\mathcal{F}_{\mathbb{C}^{-1}}$ with homogeneous boundary conditions. Use the equivalence from Exercise 6.1 to derive the a posteriori error estimate

$$\|\sigma - \tau_h\|^2_{H(\mathrm{div},\Omega)} + \|u - v_h\|^2_{H^1(\Omega)} \approx \mathcal{F}_{\mathbb{C}^{-1}}(\tau_h, v_h; f) \quad \text{for} \quad \tau_h \in \Sigma_{\mathrm{N}}(\mathcal{T}),\ v_h \in U_{\mathrm{D}}(\mathcal{T}).$$

Do τ_h and v_h have to be discrete minimisers?

(b) Extend this result to admissible Dirichlet and Neumann boundary conditions.

Hint: The exact boundary conditions have to be discretized appropriately and are prescribed explicitly in the discrete spaces $\Sigma(\mathcal{T})$ and $U(\mathcal{T})$. Use bounded extensions of the boundary data approximation errors.

Exercise 2 (Higher order least-squares FEM for linear elasticity; implementation). Implement the least-squares FEM for linear elasticity from Exercise 6.1 with conforming discretisation using the next-to-lowest-order Raviart-Thomas RT_1 and P_2 Lagrange finite elements. Compare the results of the adaptive algorithm with those from Exercise 7.2.

Literature. The following references concern the prerequisites from functional and numerical analysis required for this exercise sheet. Every reference is electronically available in the HU library and from HU intranet (e.g., eduroam).

- M. Aurada, M. Feischl, J. Kemetmüller, M. Page, and D. Praetorius. Each $H^{1/2}$-stable projection yields convergence and quasi-optimality of adaptive FEM with inhomogeneous Dirichlet data in $\mathbb{R}^d$. ESAIM Math. Model. Numer. Anal. 47(4): 1207–1235, 2013.
  - for the treatment of inhomogeneous Dirichlet boundary conditions
- P. Bringmann, C. Carstensen, and G. Starke. An adaptive least-squares FEM for linear elasticity with optimal convergence rates. SIAM J. Numer. Anal. 56(1): 428–447, 2018.
  - for the treatment of inhomogeneous Neumann boundary conditions