THE YAMABE INVARIANT

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The (conformal) Yamabe constant of a compact riemannian manifold (M, g_0) is defined as

$$Y(M, [g_0]) := \inf \int_M \mathrm{scal}^g \; dv^g$$

where the infimum runs over all metrics g of volume 1 in $[g_0]$. The (smooth) Yamabe invariant of M is then defined as

$$\sigma(M) := \sup Y(M, [g_0])$$

where the supremum runs over all conformal classes $[g_0]$ on M.

The Yamabe invariant $\sigma(M)$ is positive if and only if M carries a metric of positive scalar curvature. Despite of its simple definition, the Yamabe invariant is extremely difficult to calculate, and it is only known for very few manifolds.

I want to give an overview over the knowledge about the invariant. In particular I discuss the behavior of the Yamabe invariant under surgeries, which is joint work with Mattias Dahl and Emmanuel Humbert. Using bordism theory we can see e.g. that the Yamabe invariant of a simply connected compact spin manifold of dimension 5 is between 45 and 79.