Richard von Mises and the development of modern extreme value theory

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This lecture has the following goals:

(1) To present Richard von Mises's achievements in extreme value theory in their historical context.
(2) To provide a short history of extreme value theory and statistics since the 1920s.
(3) To discuss some of the challenges in modern extreme value theory.

In a basic course on extreme value theory one learns about the *von Mises conditions*. They are basic domain of attraction conditions ensuring the distributional convergence of affinely transformed partial maxima of an iid sequence towards a max-stable distribution. After having found (in 1923) the limiting distribution of the maxima of an iid Gaussian sequence, the so-called Gumbel distribution, in 1936, he also classified the initial distributions which are attracted to a non-degenerate limiting distribution, and he gave sufficient domain of attraction conditions. His 1936 work was based on knowledge about two other limiting distributions, M. Fréchet had found the distribution, which is named after him, in 1927. In the next year R.A. Fisher and L.H.C. Tippett published the paper which is basic for work on extreme value theory. They characterized the three possible limit distributions: Fréchet, Gumbel and Weibull. B.V. Gnedenko's 1943 paper in Annals of Mathematics *Sur la distribution limité du terme d'une série aléatoire* is the seminal paper which laid the foundations of an asymptotic theory for the extremes of an iid sequence.

The limit distributions for affinely transformed maxima of an iid sequence are called max-stable. Similar to the sum-stable distributions (the normal distribution is one of them) the max-operation acting on any finite section of an iid max-stable sequence yields the same type of distribution. It is not surprising that B.V. Gnedenko understood this problem in all its facets. He and A.N. Kolmogorov presented the solution to the stability problem for sums of independent random variables in their 1949 book which is based on a long series of results, starting from Bernoulli, Moivre, Laplace and continuing with Chinčin, Lévy, Lyapunov, Kolmogorov,.... The book summarized the knowledge about the central limit theorem for triangular schemes of independent random variables. This theory attracted much more attention than extreme value theory.

The first book treatment of extreme value theory and extreme value statistics is E.J. Gumbel's *Statistics of Extremes* from 1958. In this book, E.J. Gumbel also provides a historical account of extreme value theory since its beginnings in the 1920s. After World War II little attention was given to the study of extremes. There was little theoretical development until in 1953 a major flood in the Netherlands killed more than 1800 people. A group of mathematicians under van Dantzig gave the statistical methodology of extremal events a decisive push. Statistical analyses made a considerable contribution to the final decision making about the height of the Dutch dykes. As a consequence, extreme value theory attracted the interests of Dutch mathematicians, among them Laurens de Haan and Guus Balkema who pushed the stochastic modeling of extremes and their statistical analyses in new directions. In 1974, they, and independently J. Pickands, found the limit distribution of the excesses of an iid sequence above high thresholds, called Generalized Pareto Distribution. Their result gave a theoretical basis to the Peaks-Over-Threshold method which had been used by hydrologists for modeling extreme excesses since the 1950s.

In the 1970s, 1980s and 1990s, the foundations were laid for an extreme value theory of dependent sequences. Pioneering work was done by R. Leadbetter, H. Rootzén, S. Resnick, J. Hüsler, T. de Olivera, R.A. Davis, T. Hsing, and many others. The book by Leadbetter, Lindgren and Rootzén from 1983 was the first one that treated the extremes of stationary sequences; it essentially solved the problem for Gaussian sequences in a rather complete way. They discussed extremal clusters and how to describe them in a quantitative way. Shortly after, in 1987, S.I. Resnick published his important book *Extreme Values, Regular Variation, and Point Processes*. The focus of this book is on the relationship between the weak convergence of the point processes of the exceedances in a sample and the distributional convergence of the maxima and upper order statistics. He also provided a rigorous extreme value theory for a multivariate iid sequence.

In 1997 the book *Modelling Extremal Events for Insurance and Finance* by P. Embrechts, C. Klüppelberg and myself appeared. It was born out of the necessity that banking regulations like the Basle I treaty required to calculate very high quantiles of the return distributions of speculative assets, possibly outside the range of the data. At that time, the authors, people in academia and financial practice believed that extreme value theory might help to solve these problems. Whether this was achieved is questionable. What the book indeed achieved was that some rather theoretical results about extremes were brought to a very wide audience, including undergraduate students at many universities.

Since 1998 the extreme value community has held biannual international conferences, attracting between 300 and 500 participants, where theory and practice of extreme value theory have been discussed. Since the 1970s and 1980s the number of people interested in extreme value problems has increased by far. In 1998, H. Rootzén founded the journal Extremes which, by now, has become a successful specialized journal. The next Extreme Value Analysis Conference will be held in Delft in the end of June 2017. The last meetings have focused on topics related to stochastic processes related to extreme values and this will not change in Delft. Those topics include the important class of max-stable processes and random fields introduced by L. de Haan in 1984 which have proven utterly useful for the modeling and statistical analysis of extreme weather and climate phenomena. Multivariate extremes and dependence in time and space are still challenging problems. A positive aspect is that the extreme value community has become more open to fields like time series analysis, stochastic networks, telecommunications, branching,... Extreme value theory of high-dimensional structures is a hot topic, e.g. the extreme eigenvalues of random matrices or finding the important (extreme) components in complex stochastic systems.