

Contents

Introduction	v
I Mathematical finance in one period	1
1 Arbitrage theory	3
1.1 Assets, portfolios, and arbitrage opportunities	3
1.2 Absence of arbitrage and martingale measures	6
1.3 Derivative securities	13
1.4 Complete market models	22
1.5 Geometric characterization of arbitrage-free models	26
1.6 Contingent initial data	30
2 Preferences	43
2.1 Preference relations and their numerical representation	44
2.2 Von Neumann–Morgenstern representation	50
2.3 Expected utility	60
2.4 Uniform preferences	74
2.5 Robust preferences on asset profiles	89
2.6 Probability measures with given marginals	103
3 Optimality and equilibrium	112
3.1 Portfolio optimization and the absence of arbitrage	112
3.2 Exponential utility and relative entropy	121
3.3 Optimal contingent claims	130
3.4 Microeconomic equilibrium	141
4 Monetary measures of risk	157
4.1 Risk measures and their acceptance sets	158
4.2 Robust representation of convex risk measures	163
4.3 Convex risk measures on L^∞	175
4.4 Value at Risk	179
4.5 Measures of risk in a financial market	191
4.6 Shortfall risk	198

II	Dynamic hedging	207
5	Dynamic arbitrage theory	209
5.1	The multi-period market model	209
5.2	Arbitrage opportunities and martingale measures	213
5.3	European contingent claims	219
5.4	Complete markets	231
5.5	The binomial model	234
5.6	Convergence to the Black–Scholes price	245
6	American contingent claims	257
6.1	Hedging strategies for the seller	257
6.2	Stopping strategies for the buyer	262
6.3	Arbitrage-free prices	272
6.4	Lower Snell envelopes	277
7	Superhedging	284
7.1	\mathcal{P} -supermartingales and upper Snell envelopes	284
7.2	Uniform Doob decomposition	286
7.3	Superhedging of American and European claims	289
7.4	Superhedging with derivatives	298
8	Efficient hedging	309
8.1	Quantile hedging	309
8.2	Hedging with minimal shortfall risk	315
9	Hedging under constraints	326
9.1	Absence of arbitrage opportunities	326
9.2	Uniform Doob decomposition	333
9.3	Upper Snell envelopes	338
9.4	Superhedging and risk measures	345
10	Minimizing the hedging error	348
10.1	Local quadratic risk	348
10.2	Minimal martingale measures	358
10.3	Variance-optimal hedging	368
Appendix		375
A.1	Convexity	375
A.2	Absolutely continuous probability measures	379
A.3	The Neyman–Pearson lemma	382
A.4	The essential supremum of a family of random variables	385
A.5	Spaces of measures	386
A.6	Some functional analysis	394

Notes	399
References	403
List of symbols	413
Index	415