Faltings' Theorem

College Seminar
Summer 2015
Wednesdays 13.15-15.00 in 1.023

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The main goal of the semester is to understand some aspects of Faltings' proofs of some far–reaching finiteness theorems about abelian varieties over number fields, the highlight being the Tate conjecture, the Shafarevich conjecture, and the Mordell conjecture. There are a variety of references, including:

(A) G. Faltings. *Endlichkeitssätze für abelsche Varietäten über Zählkorpern.* We won't actually be using this.
(B) L. Szpiro (editor). *Seminaire sur les pinceaux arithmétiques: la conjecture de Mordell.* Treats everything in detail.
(C) G. Faltings and G. Wüstholz (editors). *Rational points.* Lots of detail on the representation theory side of things.

Deligne and Szpiro also have Bourbaki exposés overviewing the proof. For general background on abelian varieties, see

(D) D. Mumford. *Abelian varieties.*
(E) J. S. Milne. *Abelian varieties*, available at www.jmilne.org/math/.

Milne also treats the finiteness theorems. For the most part we'll follow C, though we'll invert the order slightly.

Outline

15.4.2015 **Overview.**
22.4.2015 **Abelian varieties over arbitrary fields.** Isogenies. Tate module. (D).
29.4.2015 **Tate module.** More on the Tate module. Weil pairing. Zarhin's trick. (D, E).
6.5.2015 **Tate and Shafarevich conjectures from finiteness.** (C §IV.2 or E §IV.2,3).
13.5.2015 **Group schemes.** Nerón models. Semistable reduction. (C §IV.3 also E §I.17).
20.5.2015 **Heights.** Isogeny formula. (C §III,IV.3).
27.5.2015 **Ramification of p-divisible groups.** (C §III).
3.6.2015 **Proof of the Tate conjecture.** (C § IV.3).
10.6.2015 **Ramification of finite group schemes.** Raynaud's theorem. (C § III.4).
17.6.2015 **Finiteness of isogeny classes.** (C §V).
24.6.2015 **Siegel moduli space: overview.** (C §I or any other reference).
1.7.2015 **Faltings height.** Hermitian line bundles and heights. (C §II or E § IV.6).
8.7.2015 **Height comparison.** (C §II).
15.7.2015 **Mordell conjecture.** Parshin's argument. Utah? (C §V.4 or E §IV.5).