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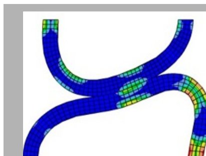
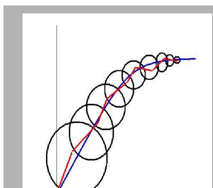
DFG-Priority Program SPP 1253 "Optimization with Partial Differential Equations".

Elliptic Mathematical Programs with Equilibrium Constraints (MPECs) in function space: optimality conditions and numerical realization.

Many phenomena in engineering, life sciences, mathematical finance or physics result in a mathematical model of variational or quasi-variational inequality type. Applications comprise contact problems (with friction) in elasticity, torsion problems in plasticity, option pricing in finance, the magnetization of superconductors or ionization problems in electrostatics. Often, one is interested in influencing the system under consideration by some control means in order to optimize a certain output quantity.


Frictional Contact - Variational Inequality of second kind

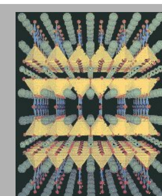
In finite dimensions the resulting optimization problems are classified as mathematical programs with equilibrium constraints (MPECs). From an optimization-theoretic point of view the treatment of MPECs is complicated by the degeneracy of the constraint set and the resulting ambiguities in associated optimality characterizations. While first and second order optimality considerations and the development of solution algorithms have reached a certain maturity in finite dimensions, the level of research is far less complete in function space.


Schematic of Inexact Path-Following

The project work concentrates on the development of a first (and, as far as possible, second order) optimality theory as well as the design and implementation of efficient solution algorithms for classes of MPECs in function space which are governed by elliptic (quasi)variational inequalities ((q)VIs). Earlier work by the applicant implies that the ambiguity when characterizing stationarity is complicated even further by the function space context, and second order theory is completely open. Moreover, the literature on control of qVIs is extremely scarce. In this respect, the project work develops new mathematical techniques for deriving and categorizing such optimality conditions in function space. The employed methodology relies on constraint relaxation to satisfy constraint qualifications, so called path-following approaches for constraints with low multiplier regularity and a subsequent asymptotic study for deriving a first order system for the original problem. In the context of qVIs, for existence results for the associated MPEC formulation, first the Mosco-convergence of the "lower-level" constraint set (which depends itself on the state) and then the stability of the solution of a qVI under data perturbations will be studied.

Since discretized MPECs result in large scale problems, tailored numerical solution techniques relying on adaptive finite element methods, semismooth Newton and multilevel techniques are developed. The semi- or non-smoothness aspect arises due to the equivalence of the first order systems to non-smooth operator equations. The primary aim of the project is to develop methods of semismooth Newton type which admit a function space analysis and exhibit therefore a stable (or even mesh independent) behavior in a discretized regime under mesh refinements. Here, the concept of slant or Newton differentiability in function space is at the core of the convergence analysis of the solution scheme. In order to accelerate the discrete algorithms multilevel techniques will be employed and adaptive methods will be used, the latter in cooperation with other groups within the SPP 1253.

The problem classes under investigation are of importance as the involved constraints, which are either qVIs or VIs of the second kind, cover a wide range of applications. The associated MPEC formulation typically aims at optimally controlling or designing the underlying system. The motivating applications for the proposal are the control of Bingham fluids and the optimization of frictional contact (i.e., both are optimal control problems for variational inequalities of the second kind) as well as the stationary magnetization of type-II superconductors, torsion problems with variable plasticity threshold and the ionization in electrostatics (i.e., the last three classes are optimal control problems for qVIs). In particular the latter three applications motivate pointwise constraints on the gradient of the state of the underlying system and a state feedback into the associated constraint set. These applications will be studied analytically and numerically within the developed theory and algorithmic schemes.



Planes of a
Type-II Superconducter