O-minimality and arithmetic aspects of Hodge theory
Fort Collins (Co) July 28-31 2025.

Abstracts

François Charles: Lefschetz theorems and tubular neighborhoods in arithmetic geometry.

I will report on joint work with Jean-Benoît Bost on the geometry of so-called formal-analytic arithmetic surfaces, which are an analogue in Arakelov geometry of formal neighborhoods of curves in surfaces. I will explain how to work with their cohomology using methods of infinite-dimensional geometry of numbers and provide applications to variations on Lefschetz hyperplane theorems.

Chris Daw: Some further progress on Zilber-Pink in $Y(1)^3$.

I will discuss work in progress with M. Orr (Manchester) and G. Papas (Weizmann) on the Zilber-Pink conjecture for $Y(1)^3$. This is known for so-called asymmetric curves by the 2012 work of Habegger-Pila. More recently, an approach known as the G-function method, has yielded further cases, namely, curves intersecting (∞, ∞, ∞) - Orr and myself -, and curves intersecting a special point in the boundary - Papas. In this work, we extend the method to deal with curves intersecting a boundary modular curve, and to give an unconditional result for points with few places of supersingular reduction.

Hélène Esnault: Algebraic flat connections and o-minimality.

We prove that an algebraic flat connection has $\mathbb{R}_{\mathrm{an,exp}}$-definable flat sections if and only if it is regular singular with unitary monodromy eigenvalues at infinity, refining previous work of Bakker-Mullane. This provides e.g. an o-minimal characterization of classical properties of the Gauss-Manin connection.

Joint work with Moritz Kerz.

Javier Fresan: On G-functions of differential order 2.

G-functions are power series that solve a linear differential equation and satisfy some growth conditions of arithmetic nature. They have close ties with arithmetic geometry because periods of pencils of algebraic varieties give rise to G-functions; the Bombieri-Dwork conjecture predicts that they all arise this way. G-functions that are solution of a differential equation of order 1 are algebraic of a rather particular shape. A rich family of G-functions of differential order 2 is given by algebraic substitutions of the classical Gauss hypergeometric series. However, I will argue that G-functions of order 2 are much richer than those. Indeed, there exist infinitely many non-equivalent differential equations of order 2 whose solutions include a G-function that is not a polynomial in algebraic substitutions of hypergeometric series. This solves Siegel's problem for G-functions, as formulated by Fischler and Rivoal, and answers a question of Krammer in connection to his counterexample to a conjecture by Dwork. (Joint work with Josh Lam and Yichen Qin).

Mark Kisin: Heights and the unramified Fontaine-Mazur conjecture.
The unramified Fontaine-Mazur conjecture is a seemingly simple statement about global p-adic Galois representations. In this talk I will explain a connection between this conjecture and the behavior of heights in the isogeny class of an Abelian variety.

Thomas Krämer: Perverse Sheaves and the Shafarevich Conjecture.
I will discuss recent work with Marco Maculan in which we prove the Shafarevich conjecture for a large class of varieties with globally generated cotangent bundle. We combine the Lawrence-Venkatesh method (à la Lawrence-Sawin) with the big monodromy theorem from our previous work with Javanpeykar and Lehn. The key ingredient is a collection of new results about perverse sheaves on irregular varieties that may be of independent interest in geometry and Hodge theory.

Wanlin Li: The story of the Ceresa cycle.
The Ceresa cycle is an algebraic 1-cycle associated to a smooth algebraic curve. It is algebraically trivial for a hyperelliptic curve and non-trivial for a very general complex curve of genus >2. Given an algebraic curve, it is an interesting question to study whether the Ceresa cycle associated to it is rationally or algebraically trivial. In this talk, I will discuss some methods and tools to study this problem. Moreover, I will discuss some recent exciting developments on the study of algebraic cycles stemmed from the interest on this cycle.

Daniel Litt: p-curvature and non-abelian cohomology.
The cohomology of a family of algebraic varieties carries a number of interrelated structures, of both Hodge-theoretic and arithmetic flavors. I'll explain joint work with Josh Lam developing analogues of some of these structures and interrelations for non-abelian cohomology—that is, the space of representations of the fundamental groups of a family of algebraic varieties, in its various incarnations. As an application, I'll explain a non-abelian version of Katz's 1972 proof of the p-curvature conjecture for Gauss-Manin connections.

Namely, let $X \to S$ be a smooth proper morphism. We show that if the p-curvature of the isomonodromy foliation on the moduli space of flat bundles on X/S vanishes, then the action of $\pi_1(S, s)$ on the space of conjugacy classes of representations of $\pi_1(X_s)$ into $GL_n(\mathbb{Z})$ factors through a finite group.

Ronnie Nagloo: On the differential approach to functional transcendence.
It is now widely recognized that the use of o-minimality, as well as other tools from algebra and geometry, to tackle Ax-Schanuel (AS) and Ax-Lindemann- Weierstrass (ALW) type theorems for automorphic functions has proven remarkably successful, particularly in addressing a range of Diophantine problems. However, in contrast to the case of the exponential function, a framework based on differential algebraic methods, avoiding o-minimal geometry, has yet to be fully developed.

In this talk, I will discuss the joint project with Blázquez-Sanz, Casale, and Freitag, which aims to use the general Ax-Schanuel type theorem we established for foliated bundles to prove AS/ALW theorems with derivatives for certain covering maps-formulated in the general setting introduced by Scanlon-and more generally for so-called geometric structures.

Ananth Shankar: a p-adic extension theorem for Shimura varieties.

Borel proved that every holomorphic map from a product of punctured unit discs to a complex Shimura varieties extends to a map from a product of discs to the Bailey-Borel compactification. In joint work with Oswal, Zhu, and Patel, we proved a p-adic version of this theorem over discretely valued fields for Shimura varieties of abelian type. I will speak about ongoing work with Bakker, Oswal, and Yao, on more general settings, including the case of non-abelian Shimura varieties.

Salim Tayou: Modularity of special cycles in orthogonal and unitary Shimura varieties.

Since the work of Jacobi and Siegel, it is well known that Theta series of quadratic lattices produce modular forms. In a vast generalization, Kudla and Millson have proved that the generating series of special cycles in orthogonal and unitary Shimura varieties are modular forms. In this talk, I will explain an extension of these results to toroidal compactifications where we prove that, when these cycles are corrected by certain boundary cycles, the resulting generating series is still a modular form in the case of divisors in orthogonal Shimura varieties and cycles of codimension up to the middle degree in unitary Shimura varieties, thereby answering a conjecture of Kudla.

The results of this talk are joint work with Philip Engel and François Greer, and joint work with François Greer.

David Urbanik: Applications of unlikely intersections to integral geometry.

We explain how a theory of infinitesimal period maps can be used to ”transfer” Hodge-theoretic unlikely intersection results from characteristic zero to positive characteristic for sufficiently large primes p. We survey several results of this type and give an idea of the methods used to prove them. Partially joint work with Ziquan Yang.