Workshop on $\mathcal{D}$-Modules and the Riemann-Hilbert Correspondence

Monday, 29 April

09:15 – 10:45: Marco Hien  
*Stokes structures and the Fourier transform I*

11:15 – 12:45: Pierre Schapira  
*Ind-Sheaves, Subanalytic sheaves and tempered holomorphic functions I*

15:00 – 16:30: Pierre Schapira  
*Ind-Sheaves, Subanalytic sheaves and tempered holomorphic functions II*

17:00 – 18:00: Francois Petit  
*Tempered subanalytic topology and the GAGA theorem*

Comparing complex algebraic and complex analytic geometry is a classical question. In the case of proper algebraic varieties, the question has been settled by Serre’s famous GAGA theorem. In the non-proper case, this theorem does not hold but some comparison results between analytic and algebraic objects have been obtained when the properness assumption is replaced by a growth condition on the analytic functions considered. In this talk, we will present an approach based on the subanalytic topology and the theory of tempered holomorphic functions which allows to obtain a partial generalization to the non-proper case of the GAGA theorem.

Tuesday, 30 April

09:15 – 10:45: Andrea D’Agnolo  
*The irregular Riemann-Hilbert correspondence I*

11:15 – 12:45: Pierre Schapira  
*Ind-Sheaves, Subanalytic sheaves and tempered holomorphic functions III*

14:15 – 15:15: Christian Sevenheck  
*Irregular Hodge filtration of hypergeometric systems*

In this talk I will explain how to obtain an explicit description of the irregular Hodge filtration, that was recently introduced in papers by Sabbah, Yu and Esnault, for certain one-dimensional $\mathcal{D}$-modules, namely the so-called classical hypergeometric systems. Irregular Hodge modules are certain mixed twistor modules for which one can define a filtration, extending the Hodge filtration of a mixed Hodge module. We use a geometric approach expressing a hypergeometric systems as a certain Gauß-Manin cohomology as well as a classical recipe of Brieskorn, Varchenko and others from Hodge theory for isolated singularities. This is joint work with Alberto Castaño Domínguez (Santiago de Compostela).

15:30 – 16:30: Claus Hertling  
*Morphic connections above F-manifolds*

17:00 – 18:00: Anna-Laura Sattelberger  
*Riemann-Hilbert for $\mathcal{D}$-modules arising from weighted projective lines*

Mirror symmetry connects the weighted projective line with a Laurent polynomial, the Landau-Ginzburg model. The quantum connection of the former appears as the Fourier-Laplace-transform of the Gauß-Manin-system of the latter, both having an irregular singularity at infinity. Following recent work of A. D’Agnolo, G. Morando, M. Hien, and C. Sabbah, we describe how to recover the Stokes data at infinity in a purely topological way.

Thursday, 02 May

09:15 – 10:45: Andrea D’Agnolo  
*The irregular Riemann-Hilbert correspondence II*
Differential systems of pure Gaussian type are meromorphic connections on the complex projective line with an irregular singularity at infinity and as such are subject to the Stokes phenomenon. Our aim is to describe the Stokes data attached to the Fourier–Laplace transform of such a system in terms of the Stokes data attached to the original system. In a work of C. Sabbah (2016), this problem has been studied using the theory of Stokes filtered local systems. Here, we use the theory of enhanced ind-sheaves and the Riemann–Hilbert correspondence of A. D’Agnolo and M. Kashiwara in order to describe the Stokes data and, under certain restrictions on the parameters, compute the Fourier–Laplace transform topologically.

Let $X \times S$ be a product of complex manifolds where $S$ is a complex line, and let $p : X \times S \to S$ be the projection. In this talk we present an overview of the construction of the relative Riemann-Hilbert functor as a right quasi-inverse to the solution functor for regular relative holonomic modules. This is a summary of joint work with Claude Sabbah which aimed to study the case of modules underlying a mixed twistor $\mathcal{D}$-module. If the dimension of $X$ is 1, the situation is simpler to deal with, and we proved that, in a generic sense, any relative holonomic module admits a coherent restriction to any divisor $Y \times S$, which, in turn, allowed us to prove that, in a generic sense, the Riemann-Hilbert functor is also a left quasi-inverse functor. As a consequence of work in progress with Luisa Fiorot, the condition of genericity can be withdrawn.

This talk is based on a joint work with Teresa Monteiro Fernandes concerning the study of relative holonomic $\mathcal{D}$-modules. Contrary to the absolute case, the standard $t$-structure on holonomic $\mathcal{D}$-modules is not preserved by duality and hence the solution functor is no longer $t$-exact with respect to the canonical, resp. middle-perverse, $t$-structure. We provide an explicit description of these dual $t$-structures and we prove the Solution functor is exact with respect to the canonical, resp. dual middle-perverse, $t$-structure. We present a strategy to prove the Relative Riemann-Hilbert correspondence (regular case) for $X \times S$ relative to $S$ where $S$ is a complex line and $X$ is a complex manifold.