Stochastik II
Wintersemester 2018/19
Humboldt-Universität zu Berlin
Prof. Dr. Dörte Kreher

Exercise Sheet 2

Problem 1 (4 Points)
Let T_1, T_2 be independent, identically distributed random variables on $(\Omega, \mathcal{F}, \mathbb{P})$ having an exponential law with parameter $\lambda > 0$. We set $X := \min\{T_1, T_2\}$.

a) Show that $\mathbb{E}(X|T_1) = \frac{1}{\lambda}(1 - e^{-\lambda T_1})$.

b) Prove that the best linear estimator $\hat{X}$ of X based on T_1 is given by $\hat{X} = \frac{1}{4}(T_1 + \frac{1}{\lambda})$.

Problem 2 (4 Points)
Let $(X_i)_{i\in\mathbb{N}} \subset L^2(\Omega, \mathcal{F}, \mathbb{P})$ be a family of i.i.d. random variables being Bernoulli distributed with parameter $p \in (0, 1)$. Let N be a Poisson distributed random variable with parameter $\lambda > 0$, independent of the family $(X_i)_{i\in\mathbb{N}}$. Put

$$S := \sum_{i=1}^{N} X_i \quad \text{with} \quad \sum_{i=1}^{0} X_i := 0.$$

Determine $\mathbb{E}(S|N)$ and $\mathbb{E}(N|S)$.

Problem 3 (4 Points)
Let X, Y be two independent real-valued random variables on $(\Omega, \mathcal{F}, \mathbb{P})$. Suppose that $Y \sim N(0, 1)$.

a) Show that the following properties are equivalent:
   (i) $\mathbb{E}(e^{X^2/2}) < \infty$,
   (ii) $\mathbb{E}(e^{XY}) < \infty$,
   (iii) $\mathbb{E}(e^{|XY|}) < \infty$.

b) Show that if $\mathbb{E}(e^{X^2/2}) < \infty$, then $\mathbb{E}(e^{XY}|X) \geq 1$ almost surely.

Problem 4 (4 Points)

a) Let $(X, Y) \sim N(\mu, \Sigma)$ with $\mu = \begin{pmatrix} 17 \\ 42 \end{pmatrix}$ and $\Sigma = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$. Compute $\mathbb{E}(X|Y)$.

b) Let U, V, W be independent, standard normal distributed random variables. Show that

$$\frac{U + VW}{\sqrt{1 + W^2}} \sim N(0, 1).$$

Please hand in your solutions on Monday, 5 November, 2018 at 9:15 am.