Stochastik II
Wintersemester 2018/19
Humboldt-Universität zu Berlin
Prof. Dr. Dörte Kreher

Exercise Sheet 9

Problem 1 (4 Points)
Compute the invariant distributions for the following two Markov chains with state space E and transition matrix $P = (p_{ij})$.

a) Ehrenfest model: $E = \{0, 1, \ldots, N\}$ for some $N \geq 2$ and

$$p_{i,i+1} = 1 - \frac{i}{N} \quad \text{for } i = 0, 1, \ldots, N-1, \qquad p_{i,i-1} = \frac{i}{N} \quad \text{for } i = 1, 2, \ldots, N$$

b) $E = \mathbb{N}_0$ and for all $i, j \in \mathbb{N}_0$:

$$p_{ij} = \begin{cases} \frac{1}{i+2} & : j = 0, 1, \ldots, i+1 \\ 0 & : \text{else} \end{cases}$$

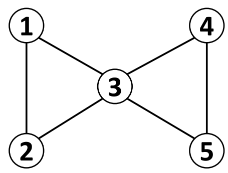
Problem 2 (4 Points)
A particle performs a random walk on the vertex set of a connected graph G with $\eta \in \mathbb{N}$ edges, which has no loops or multiple edges between any two vertices. At each stage it moves to a neighbour of its current position, each such neighbour being chosen with equal probability.

a) Show that the invariant distribution is given by $\pi_v = \frac{d_v}{2\eta}$, where d_v is the degree of vertex v, i.e. the number of its neighbours.

b) A chess piece performs a random walk on a chessboard; at each step it is equally likely to make any one of the available moves. What is the mean recurrence time to a corner square if the piece is a king / rook / knight / bishop / queen?

Problem 3 (4 Points)

A particle performs a random walk on the graph shown on the right. From any vertex its next step is equally likely to be to any neighbouring vertex. Initially it is in vertex 1. Find the expected value of:



a) the time of its first return to 1,

b) the number of its visits to 4 before returning to 1,

c) the time of its first return to 1, given no prior visit to 5,

d) the number of its visits to 4 before returning to 1, given no prior visits to 5.

Problem 4 (4 Points)
A random sequence of non-negative integers $(Z_n)_{n\in\mathbb{N}_0}$ is obtained by fixing some $Z_0, Z_1 \in \mathbb{N}_0$ and iteratively defining Z_{n+1} randomly with equal probability as either the sum or the absolute difference of Z_{n-1} and Z_n for $n \geq 1$.

a) Is $Z = (Z_n)_{n\in\mathbb{N}_0}$ a Markov chain?

b) Show that $X_n := (Z_{n-1}, Z_n),\ n \in \mathbb{N}$ is a Markov chain and find the probability that Z reaches 3 before its first return to 0, if started from $Z_0 = 0,\ Z_1 = 1$.

c) Compute the probability that X hits $(1, 1)$ when starting from $(1, 2)$.

d) Show that X is transient and that $Z_n \to \infty$ almost surely.

Please hand in your solutions on Monday, 7 January, 2019 at 9:15 am.