

Extending SCIP for solving MIQCPs

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Mixed-Integer Quadratically Constrained Programming (MIQCP)

We consider optimization problems of the form

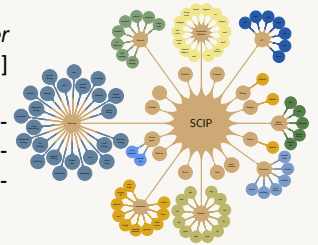
$$\begin{aligned} & \text{minimize} && b_0^T x + c_0 \\ & \text{subject to} && x^T A_j x + b_j^T x + c_j \leq 0 \quad j = 1, \dots, m \\ & && x_i \in \mathbb{Z} \quad \forall i \in I, \end{aligned}$$

where $A_j \in \mathbb{Q}^{n \times n}$, $b_j \in \mathbb{Q}^n$, $c_j \in \mathbb{Q}$, $j = 0, \dots, m$, and $I \subseteq \{1, \dots, n\}$.

In general, A_j does not need to be positive semidefinite.

Algorithm: LP-based branch-and-cut

We extend the *Constraint Integer Programming* framework SCIP [1] by MINLP-specific plugins. For nonlinear constraints, we generate a suitable linear outer approximation and apply domain propagation and primal heuristics.



Currently implemented [5]

- ▷ reformulation for products with binary variables
- ▷ recognition of convex quadratic functions
- ▷ separation for convex and nonconvex quad. constraints
- ▷ domain propagation for quadratic constraints
- ▷ handling of second-order-cone (SOC) constraints
- ▷ local search heuristic: fix integers, solve *sub-QCP* locally
- ▷ interfaces to GAMS, MPS, and ZIMPL

Nonlinear RENS Heuristic

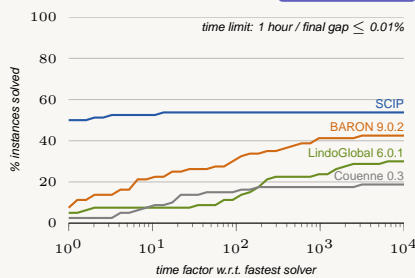
Relaxation Enforced Neighborhood Search heuristic [2]: fix integer variables that take integral value in optimal solution of LP relaxation and solve remaining *sub-MIQCP*

Undercover Heuristic [3]

construct (and solve) *sub-MIP* by fixing as few nonlinear variables as possible to their value in an optimal solution of the LP or NLP relaxation

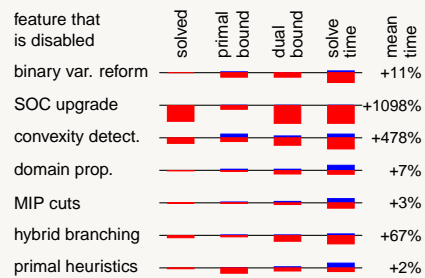
Computational Results

Benchmark

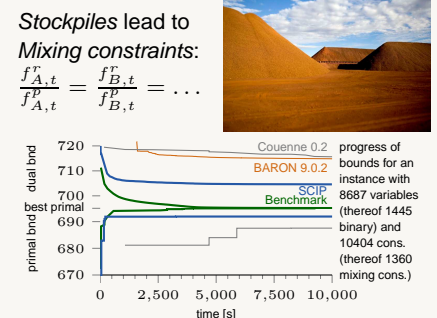


80 MIQCP benchmark instances from MINLPlib, H. Mittelmann, J.P. Vielma

Impact of Single Components



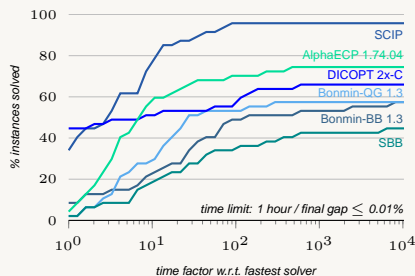
Application: Mine Prod. Scheduling [6]



Beyond MIQCP

Convex MINLP

QG-like algorithm (gradient-based cuts)



47 convex MINLP benchmark instances from P. Bonami, M. Kiliç, J. Linderoth (2009)

Pseudo-Boolean Optimization [4]

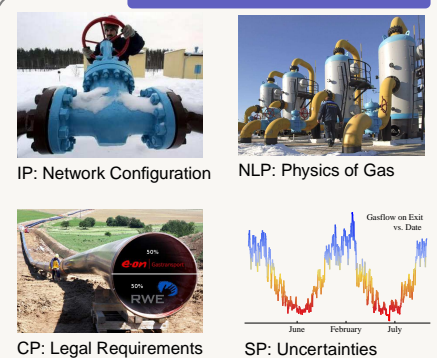
Pseudo-Boolean Constraints:

$$\sum_{j=1}^{t_i} a_{ij} \cdot \prod_{k \in I_{ij}} x_k \cdot \prod_{k \in \bar{I}_{ij}} (1 - x_k) \geq b_i$$

$$x_k \in \{0, 1\}, k \in I_{ij} \cup \bar{I}_{ij}$$

- ▷ replace multiplications by new variables and *AND-constraints*
- ▷ specialised separation and propagation for *AND-constraints* (small initial relax., separate strong ineq., always propagate)
- ▷ Winner in 3 out of 4 categories at Pseudo-Boolean Evaluation 2009

Application: Gas Transport



References

- [1] T. Achterberg. SCIP: Solving Constraint Integer Programs, *Mathematical Programming Computation* 1 (2009), pp. 1–41
- [2] T. Berthold. RENS – Relaxation Enforced Neighborhood Search, ZIB-Report 07-28
- [3] T. Berthold, A. Gleixner. Undercover – a primal heuristic for MINLP based on sub-MIPs generated by set covering, ZIB-Report 09-40
- [4] T. Berthold, S. Heinz, M.E. Pfetsch. Nonlinear pseudo-Boolean optimization: relaxation or propagation?, *Proc. of SAT 2009*, pp. 441–446
- [5] T. Berthold, S. Heinz, S. Vigerske. Extending a CIP framework to solve MIQCPs, ZIB-Report 09-23
- [6] A. Bley, A. Gleixner, T. Koch, S. Vigerske. Comparing MIQCP solvers to a specialised algorithm for mine production scheduling, ZIB-Report 09-32

Cooperations

