# Problems for BMS Basic Course "Commutative Algebra"

Hand in whenever you like at room 2.304

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**Please sign each sheet of paper with your name and student ID**

Let $A, B$ objects of an abelian category $\mathfrak{C}$ and $\varphi : A \longrightarrow B$ be a morphism (which is, as we know by now, just a 4-term exact sequence

$$0 \longrightarrow \ker(\varphi) \longrightarrow A \longrightarrow B \longrightarrow \mathrm{coker}(\varphi) \longrightarrow 0.)$$

$P \in \mathrm{Ob}(\mathfrak{C})$ is called a *pumpkin* of $\varphi$ if there are morphisms $j : A \longrightarrow P$, $\pi : P \longrightarrow B$ such that:

(i) $\varphi = \pi \circ j$.

(ii) $j$ is a monomorphism, $\pi$ is an epimorphism.

(iii) There are induced exact sequences

$$0 \longrightarrow \ker(\varphi) \longrightarrow P \longrightarrow B \longrightarrow 0$$

and

$$0 \longrightarrow A \longrightarrow P \longrightarrow \mathrm{coker}(\varphi) \longrightarrow 0.$$

a) For given $A, B, \varphi$, is there always a pumpkin?

b) If it exists, is a pumpkin unique up to isomorphism?

c) Find necessary and sufficient conditions for the existence and uniqueness of pumpkins.